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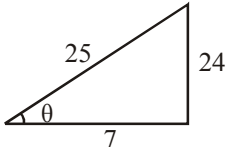
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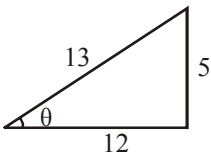
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BASIC MATHS (TRIGONOMETRY) : RACE # 01

SOLUTIONS

1. (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
 (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
 (e) $\frac{2\pi}{3}$ (f) $\frac{3\pi}{4}$
 (g) $\frac{7\pi}{6}$ (h) $\frac{3\pi}{2}$ (i) $\frac{7\pi}{4}$
2. (a) 45° (b) 30°
 (c) 60° (d) 135° (e) 210°
 (f) 225° (g) 450°
 (h) 315° (i) 300°
3. (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
 (c) $-\frac{1}{2}$ (d) 1 (e) $-\frac{1}{2}$
 (f) $-\frac{2}{\sqrt{3}}$ (g) $-\frac{1}{\sqrt{2}}$ (h) $-\frac{\sqrt{3}}{2}$
4. (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
 (c) $\sqrt{3}$ (d) 0 (e) -1
 (f) $\frac{1}{2}$ (g) 0 (h) -1
 (i) 1 (j) -1 (k) 0
5. (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$
 (d) 0 (e) $\frac{1}{2}$ (f) $\frac{1}{2}$
6. (a) $-\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-\sqrt{3}}{2}$ (d) -1
7. (a) $\cos \theta = \frac{7}{25}$
- 
- $\sin \theta = \frac{24}{25}$ $\operatorname{cosec} \theta = \frac{25}{24}$ $\sec \theta = \frac{25}{7}$
 $\tan \theta = \frac{24}{7}$ $\cot \theta = \frac{7}{24}$

- 
- (b) $\sin \theta = \frac{5}{13}$
 $\cos \theta = \frac{12}{13}$ $\sec \theta = \frac{13}{12}$ $\operatorname{cosec} \theta = \frac{13}{5}$
 $\tan \theta = \frac{5}{12}$ $\cot \theta = \frac{12}{5}$
8. (a) $\cos 75^\circ = \cos (45^\circ + 30^\circ)$
 $= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$
 $= \frac{\sqrt{3}-1}{2\sqrt{2}}$
 (b) $\sin 15^\circ = \sin (90^\circ - 75^\circ) = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$
 (c) $\sin 75^\circ = \sin (45^\circ + 30^\circ)$
 $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
 $= \frac{\sqrt{3}+1}{2\sqrt{2}}$
 (d) $\cos 105^\circ = \cos (90^\circ + 15^\circ)$
 $= -\sin 15^\circ = \frac{1-\sqrt{3}}{2\sqrt{2}}$
9. Distance from x-axis is
 $y = r \sin \theta$
 $y_A = 10 \sin 60^\circ \Rightarrow y_A = 5\sqrt{3}$
 $y_B = 10 \sin 143^\circ \Rightarrow y_B = 6$
 Distance from y-axis is
 $x = r \cos \theta$
 $x_A = 10 \cos 60^\circ \Rightarrow x_A = 5$
 $x_B = 10 \cos 143^\circ \Rightarrow x_B = -8$
11. $2(\sin 15^\circ + \sin 75^\circ)^2$
 $\Rightarrow 2(\sin 15^\circ + \cos 15^\circ)^2$ ($\because \sin \theta = \cos(\pi/2 - \theta)$)
 $\Rightarrow 2[\sin^2 15^\circ + \cos^2 15^\circ + 2\sin 15^\circ \cos 15^\circ]$
 $\Rightarrow 2[1 + \sin 30^\circ]$ ($\because 2\sin \theta \cos \theta = \sin 2\theta$)
 $\Rightarrow 3$
12. $5(\sin 100^\circ \cos 27^\circ + \sin 27^\circ \cos 100^\circ)$
 $\Rightarrow 5 \sin (100^\circ + 27^\circ)$
 $\Rightarrow 5 \sin (90^\circ + 37^\circ)$
 $\Rightarrow 5 \cos 37^\circ$
 $\Rightarrow 4$

13. If $\theta \leq 5^\circ$

$\tan\theta \approx \theta$ (θ should be in radian)

$$\frac{h}{1} = 1.8^\circ \times \frac{\pi}{180^\circ}$$

$$h = \frac{\pi}{100} \text{ m}$$

14. $4 - 2\cos\theta$

$$\Rightarrow -1 \leq \cos\theta \leq 1$$

Here

For maximum value

use ($\cos\theta = -1$)

$$\Rightarrow 4 - 2(-1)$$

$$\Rightarrow 6$$

For minimum value

use ($\cos\theta = 1$)

$$\Rightarrow 4 - 2(1) \Rightarrow 2$$

15. For $\triangle BCO$

$$\tan 60^\circ = \frac{y}{x}$$

$$x = \left(\frac{y}{\sqrt{3}} \right) \quad \dots(1)$$

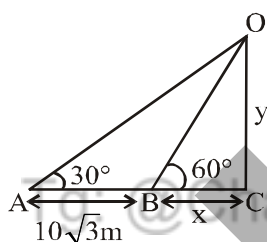
For $\triangle ACO$

$$\tan 30^\circ = \frac{y}{(x + 10\sqrt{3})}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{\left(\frac{y}{\sqrt{3}} + 10\sqrt{3} \right)}$$

$$y + 30 = 3y$$

$$y = 15\text{m}$$



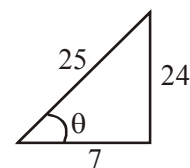
16. $\tan\theta = \frac{24}{7}$

$\tan\theta$ is $\oplus$ ve

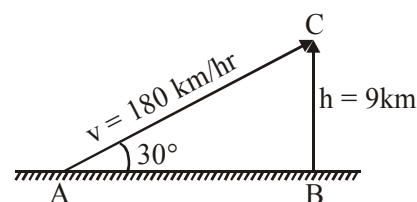
& $\sin\theta$ is $\ominus$ ve

it means III-quadrant

$$\cos\theta = -\frac{7}{25}$$



19.



In $\triangle ABC$

$$\sin 30^\circ = \frac{h}{AC}$$

$$\frac{1}{2} = \frac{9}{AC}$$

$$AC = 18\text{km}$$

time taken by plane

$$18 = (180)t$$

$$t = \frac{1}{10} \text{ hr}$$

$$t = 6 \text{ min.}$$

BASIC MATHS (DIFFERENTIATION) : RACE # 02

SOLUTIONS

1. $y = \frac{4}{3}x - 4$

(i) $\frac{dy}{dx} = \frac{4}{3} = \tan 53^\circ$

(ii) $\frac{dx}{dy} = \frac{3}{4} = \tan 37^\circ$

(iii) x-intercept means $(y = 0) \Rightarrow x = 3$

(iv) length = distance

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d_{AB} = 5$$

2. (i) True (ii) True (iii) True (iv) False

3. $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi \left(3r^2 \frac{dr}{dt} \right) \quad \left(\because \frac{dr}{dt} = \frac{2}{\pi} \right)$$

$$\frac{dv}{dt} = \frac{4}{3}\pi 3 \times (0.5)^2 \times \left(\frac{2}{\pi} \right) \Rightarrow \frac{dv}{dt} = 2$$

4. $y = \sin 3x + \frac{4}{3}\cos 3x$

$$\frac{dy}{dx} = 3\cos 3x - 4\sin 3x$$

Maximum value for function $a \sin \theta + b \cos \theta$

is $\sqrt{a^2 + b^2}$ so $\left(\frac{dy}{dx} \right)_{\text{MAX}} = 5$

5. (i) $y = x^{-1/3}$

$$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{x^{-4/3}}{3}$$

(ii) $x = t^6$

$$\frac{dx}{dt} = 6t^{6-1}$$

$$\frac{dx}{dt} = 6t^5$$

(iii) $z = u^3$

$$\frac{dz}{du} = 3u^2$$

(iv) $f = r^2 + r^{1/3}$

$$\frac{df}{dr} = 2r + \frac{r^{-2/3}}{3}$$

(v) $X = 2t^3 - 3t^2 + 5t - 6$

$$\frac{dx}{dt} = 6t^2 - 6t + 5$$

(vi) $y = 3x^2$

$$\frac{dy}{dx} = 6x$$

(vii) $y = x \ln x$

apply $y = uv$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

so

$$\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \ln x$$

$$= 1 + \ln x$$

(viii) $y = x \sin x$ (apply $y = u \cdot v$)

$$\frac{dy}{dx} = \sin x + x \cos x$$

(ix) $y = \sin x \cos x$ (apply $y = u \cdot v$)

$$\frac{dy}{dx} = \cos^2 x - \sin^2 x$$

(x) $y = \sin^2 x + \cos^2 x$

$$y = 1 \rightarrow \frac{dy}{dx} = 0$$

(xi) $y = \cos 2x$

Let $t = 2x \rightarrow \frac{dt}{dx} = 2$

so $y = \cos t$

$$\frac{dy}{dx} = -\sin t \cdot \frac{dt}{dx}$$

$$\frac{dy}{dx} = -2 \sin 2x$$

(xii) $v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dr} = 4\pi r^2$$

(xiii) $s = 4\pi r^2$

$$\frac{ds}{dr} = 8\pi r$$

6. $\frac{d}{dx} \cos(\sqrt{x})$

Let $t = \sqrt{x} \rightarrow \frac{dt}{dx} = \frac{1}{2\sqrt{x}}$

so $\frac{d}{dx}(\cos t)$

$$\Rightarrow -\sin t \cdot \frac{dt}{dx} \Rightarrow -\frac{1}{2\sqrt{x}} \times \sin(\sqrt{x})$$

7. $y = x^2 - 2\sqrt{x}$

$$\frac{dy}{dx} = 2x - 2\left(\frac{1}{2\sqrt{x}}\right)$$

$$\left(\frac{dy}{dx}\right)_{x=1} = 2 - 1 \Rightarrow 1$$

8. $h(t) = -16t^2 + 256t$

For maximum value of h

$$\frac{dh}{dt} = -32t + 256 = 0$$

$$t = 8 \text{ sec}$$

9. $y = \ln(\cos x)$

$$\text{Let } t = \cos x \rightarrow \frac{dt}{dx} = -\sin x$$

$$\text{so } y = \ln t$$

$$\frac{dy}{dx} = \frac{1}{t} \cdot \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos x} \times (-\sin x) = -\tan x$$

$$\text{at } x = \frac{3\pi}{4}$$

$$\left(\frac{dy}{dx}\right)_{x=3\pi/4} = -\tan 3\pi/4 = 1$$

10. Given $\frac{dr}{dt} = 0.5 \text{ m/s}$

$$s = \pi r^2$$

$$\frac{ds}{dt} = 2\pi r \frac{dr}{dt}$$

$$\left(\frac{ds}{dt}\right)_{r=4/\pi} = 2\pi(4/\pi)(0.5) = 4$$

11. $q = 2t^2 + 3t + 1$

$$\frac{dq}{dt} = 4t + 3$$

$$i_{t=5} = 4(5) + 3$$

$$i_{t=5} = 23$$

12. (i) $y = x^2 + 5$

$$\frac{dy}{dx} = 2x$$

(ii) $y = 2e^3$

$$\frac{dy}{dx} = 0$$

(iii) $y = (x + 5)^{-1/2}$

$$\text{Let } t = (x + 5) \rightarrow \frac{dt}{dx} = 1$$

$$y = t^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2} t^{-3/2}$$

$$\frac{dy}{dx} = -\frac{1}{2} (x + 5)^{-3/2}$$

(iv) $y = 5x^{3/2}$

$$\frac{dy}{dx} = \frac{15}{2} x^{1/2}$$

(v) $y = 4x^4 + 2x^3 + \frac{5}{x} + 9$

$$\frac{dy}{dx} = 16x^3 + 6x^2 - \frac{5}{x^2} \quad \left(\frac{dx^n}{dx} = nx^{n-1}\right)$$

13. (1) $y = \cos x^3$

$$\text{Let } t = x^3 \rightarrow \frac{dt}{dx} = 3x^2$$

$$y = \cos t$$

$$\frac{dy}{dx} = -\sin t \left(\frac{dt}{dx}\right)$$

$$= -3x^2 \sin x^3$$

(2) $y = \sin(x/2)$

$$\text{Let } t = x/2 \rightarrow \frac{dt}{dx} = \frac{1}{2}$$

$$y = \sin t$$

$$\frac{dy}{dx} = \cos t \frac{dt}{dx}$$

$$= \frac{1}{2} \cos \frac{x}{2}$$

(3) $y = \log_e 2x$

$$\text{Let } t = 2x \rightarrow \frac{dt}{dx} = 2$$

$$y = \log_e t$$

$$\frac{dy}{dx} = \frac{1}{t} \left(\frac{dt}{dx}\right)$$

$$\frac{dy}{dx} = \frac{1}{2x} (2) = \frac{1}{x}$$

(4) $y = e^{-x}$

$$\text{Let } t = -x \rightarrow \frac{dt}{dx} = -1$$

$$\frac{dy}{dx} = e^t \frac{dt}{dx}$$

$$= (-1)e^{-x} = -e^{-x}$$

14. $y = e^x \sin x$

$$\frac{dy}{dx} = e^x \cos x + e^x \sin x \quad \left[\begin{array}{l} \text{by using} \\ (y=uv) \end{array} \right]$$

$$\frac{dy}{dx} = e^x (\cos x + \sin x)$$

15. $x = 2 + 5t + 7t^2$

(1) $v = 5 + 14t$

(2) $u_{t=0} = 5 + 14(0)$

(3) $u_{t=2} = 5 + 14(2)$
 $= 33 \text{ m/s}$

(4) $a = 14 \text{ m/s}^2$

$$\left(v = \frac{dx}{dt} \right)$$

$$\left(a = \frac{dv}{dt} \right)$$

16. $A = 5t^2 + 4t$

$$\frac{dA}{dt} = 10t + 4$$

$$\left(\frac{dA}{dt} \right)_{t=3} = 10(3) + 4$$

 $= 34 \text{ m}^2/\text{s}$

17. $v = (4t^2 + 3t + 1) \text{ m/s}$

$a = 8t + 3$

$a_{t=1} = 11 \text{ m/s}^2$

18. $s = 12t + 3t^2 - 2t^3$

$v = 12 + 6t - 6t^2$

at $t = 0$

$v = 12 \text{ m/s}$

19. $x = 2 - 5t + 6t^2$

$v = -5 + 12t$

$a_{at \ t=0} = -5 \text{ m/s}$

20. $x = 2 - 5t + 6t^2$

$v = -5 + 12t$

$a = 12 \text{ m/s}^2$

21. $v = (t + 2)(t + 3)$

$v = t^2 + 5t + 6$

$a = 2t + 5$

at $t = 1 \text{ sec}$

$a_{t=1} = 7 \text{ m/s}^2$

22. $y = \log_e x + \sin x + e^x$

$$\frac{dy}{dx} = \frac{1}{x} + \cos x + e^x$$

23. $\frac{d}{dx}(e^{100}) = 0$

24. $\frac{d}{dx}(\sin 120^\circ) = 0$

25. $y = x^3 \cos x$

apply $(y = u \cdot v)$

$$\frac{dy}{dx} = 3x^2 \cos x - x^3 \sin x$$

 $= x^2(3 \cos x - x \sin x)$

26. $v = t^2 - 4t + 10^5$

$a = 2t - 4$

at $t = 1 \text{ sec}$

$a = -2 \text{ m/s}^2$

27. $y = \sin x + \cos x$

$$\frac{dy}{dx} = \cos x - \sin x$$

$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

 $= -(\sin x + \cos x)$

28. $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi(3r^2) \frac{dr}{dt}$$

$$= 4\pi r^2 \left(\frac{dr}{dt} \right)$$

29. $y = \frac{10}{\sin x + \sqrt{3} \cos x}$

Assume $t = \sin x + \sqrt{3} \cos x$

for $y_{\min}$, t should be maximum

$$t_{\max} = \sqrt{1 + (\sqrt{3})^2}$$

$t_{\max} = 2$

$$y_{\min} = \frac{10}{2}$$

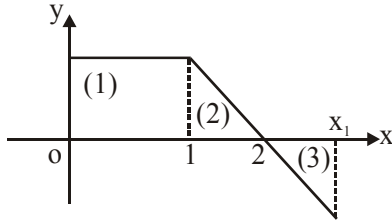
$y_{\min} = 5$

BASIC MATHS (INTEGRATION) : RACE # 03

SOLUTIONS

1. $\int_0^{x_1} y dx = 5$

means area under the curve should be 5



Area (1) = 5

so

Area (2) = Area (3) $\Rightarrow x_1 = 3$

2. (a) $\int \frac{1}{\sqrt{x}} dx$

$\Rightarrow \int x^{-1/2} dx$

$\Rightarrow \frac{x^{1/2}}{1/2} + c$

$\Rightarrow 2\sqrt{x} + c$

(b) $\int \frac{dx}{(2x+3)}$

$\Rightarrow$ Let $t = (2x+3) \rightarrow \frac{dt}{dx} = 2$

or $dx = \frac{dt}{2}$

$\Rightarrow \int \frac{dt}{2(t)}$

$\Rightarrow \frac{1}{2} \log_e t + c \Rightarrow \frac{1}{2} \log_e (2x+3) + c$

(c) $\int \sin x \cos x dx$

$\Rightarrow \frac{1}{2} \int (2 \sin x \cos x) dx$

$\Rightarrow \frac{1}{2} \int \sin 2x dx$

Let $t = 2x \rightarrow \frac{dt}{dx} = 2$

or $dx = \frac{dt}{2}$

So

$= \frac{1}{4} \int \sin t dt$

$\Rightarrow -\frac{1}{4} \cos t + c$

$\Rightarrow -\frac{1}{4} \cos 2x + c$

3. (a) $\int_0^3 (x^2 + 1) dx$

$\Rightarrow \left[\frac{x^3}{3} + x \right]_0^3$

$\Rightarrow 9 + 3 = 12$

(b) $\int_2^3 (x^3 - 4x^2 + 5x - 10) dx$

$\Rightarrow \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{5x^2}{2} - 10x \right]_2^3$

$\Rightarrow \left[\frac{81}{4} - \frac{4(27)}{3} + \frac{5(9)}{2} - 10(3) \right] - \left[\frac{16}{4} - \frac{4 \times 8}{3} + \frac{5 \times 4}{2} - 10(2) \right]$

$= -\frac{79}{12}$

(c) $\int_0^{\pi/4} (\cos x - \sin x) dx$

$\Rightarrow [\sin x + \cos x]_0^{\pi/4}$

$\Rightarrow \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - 1 \right]$

$\Rightarrow \sqrt{2} - 1$

4. (1) $\int x^{-3/2} dx$

$$\Rightarrow \frac{x^{-3/2+1}}{\left(-\frac{3}{2}+1\right)} + c$$

$$\Rightarrow -\frac{2}{\sqrt{x}} + c$$

(2) $\int \sin 60^\circ dx$

$$\Rightarrow \sin 60^\circ \int dx$$

$$\Rightarrow \frac{\sqrt{3}}{2} x + c$$

(3) $\int \frac{1}{10x} dx$

$$\Rightarrow \frac{1}{10} \int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{10} \log_e x + c$$

(4) $\int (2x^3 - x^2 + 1) dx$

$$\Rightarrow 2\left(\frac{x^4}{4}\right) - \frac{x^3}{3} + x + c$$

$$\Rightarrow \frac{x^4}{2} - \frac{x^3}{3} + x + c$$

5. $\int_0^2 3x^2 dx + \int_0^{\pi/2} \sin x dx$

$$\Rightarrow 3\left[\frac{x^3}{3}\right]_0^2 - [\cos x]_0^{\pi/2}$$

$$\Rightarrow 8 - (0 - 1)$$

$$\Rightarrow 9$$

6. $v = 3t^2 - 2t$

$$\frac{dx}{dt} = 3t^2 - 2t$$

$$\int dx = \int (3t^2 - 2t) dt$$

When $t = 0 \longrightarrow x = 0$

$t = 2 \longrightarrow x = ?$

$$\int_0^x dx = \int_0^2 (3t^2 - 2t) dt$$

$$x = \left[t^3 - t^2\right]_0^2$$

$$x = 4$$

7. (i) $\int \left(t - \frac{1}{t}\right)^2 dt$

$$\Rightarrow \int \left(t^2 + \frac{1}{t^2} - 2\right) dt$$

$$\Rightarrow \left[\frac{t^3}{3} - \frac{1}{t} - 2t\right] + c$$

(ii) $\int \sin(10t - 50) dt$

Let $x = (10t - 50) \longrightarrow \frac{dx}{dt} = 10$

or $dt = \frac{dx}{10}$

$$\Rightarrow \int \sin x \frac{dx}{10}$$

$$\Rightarrow \frac{1}{10} (-\cos x) + c$$

$$\Rightarrow -\frac{\cos(10t - 50)}{10} + c$$

(iii) $\int e^{(100t+6)} dt$

Let $x = 100t + 6 \longrightarrow \frac{dx}{dt} = 100$

or $dt = \frac{dx}{100}$

$$\int e^x \frac{dx}{100}$$

$$\Rightarrow \frac{1}{100} e^x + c \Rightarrow \frac{1}{100} e^{(100t+6)} + c$$

8. $a = 2t + 5$

$$\frac{dv}{dt} = 2t + 5$$

$$\int_0^v dv = \int_0^2 (2t + 5) dt$$

$$v = [t^2 + 5t]_0^2$$

$$v = 4 + 10 \Rightarrow v = 14 \text{ m/s}$$

9. $\int_2^4 \frac{dx}{x}$

$$\Rightarrow [\log_e x]_2^4$$

$$\Rightarrow \log_e 4 - \log_e 2$$

$$\Rightarrow \log_e \frac{4}{2}$$

$$\Rightarrow \log_e 2$$

10. $y = \sin x$

$$\text{Area} = \int y dx$$

$$= \int_0^{\pi/2} \sin x dx$$

$$\Rightarrow -[\cos x]_0^{\pi/2}$$

$$\Rightarrow -\left[\cos \frac{\pi}{2} - \cos 0\right]$$

$$\Rightarrow 1$$

Tg: @Chalnaayaaar

CO-ORDINATE GEOMETRY & GRAPHS : RACE 04

SOLUTIONS

6. $\sqrt{3}y = 3x + 4$

$$y = \frac{3}{\sqrt{3}}x + \frac{4}{\sqrt{3}}$$

$$y = \sqrt{3}x + \frac{4}{\sqrt{3}}$$

slope (m) = $\sqrt{3}$

7. $m = \frac{\Delta y}{\Delta x} = \frac{-8}{6}$

$$m = -\frac{4}{3}$$

$$y = -\frac{4}{3}x + 8$$

$$3y + 4x = 24$$

9. $y = 2x^2$

$$\frac{dy}{dx} = 4x$$

Slope of graph at P(1, 2)
so $x = 1$

$$\left(\frac{dy}{dx}\right)_P = 4$$

14. $y = 2 + ae^{-x}$
put $y = 6$ at $x = 0$
 $6 = 2 + ae^{-0}$
 $a = 4$

15. $\frac{dy}{dx} = x^2$

graph between $\frac{dy}{dx}$ and x will be parabola.

16. Here, $b = 6$

$$\text{Area of } \triangle OAB = \frac{1}{2}ab$$

$$72 = \frac{1}{2}(a)(6)$$

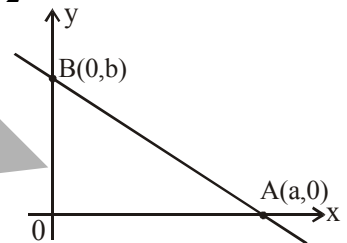
$$a = 24$$

Slope of line AB

$$m = -\frac{1}{4}$$

so equation of line AB

$$y = -\frac{1}{4}x + 6$$



BASIC MATHS (ZERO LEVEL) & ALGEBRA : RACE 05

SOLUTIONS

15. In $\triangle ACD$

$\angle CDA = 90^\circ$ and $CA = 2OA$

$AC^2 = CD^2 + DA^2$ (by Pythagoras theorem)

$$(2OA)^2 = a^2 + a^2$$

$$4OA^2 = 2a^2$$

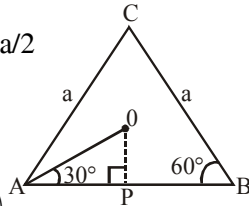
$$OA = \frac{a}{\sqrt{2}}$$

16. In $\triangle AOP$

$\angle APO = 90^\circ$ and $AP = a/2$

$$\cos 30^\circ = \frac{AP}{AO}$$

$$\frac{\sqrt{3}}{2} = \frac{a/2}{AO} \Rightarrow \left(AO = \frac{a}{\sqrt{3}} \right)$$



21. Given $4\sqrt{6} \times 3\sqrt{24}$

$$\Rightarrow 4 \times 3 \times \sqrt{6} \sqrt{24}$$

$$= 12 \times \sqrt{144}$$

$$\Rightarrow 12 \times 12 = 144$$

22. Longest bar = diagonal of room

$$= \sqrt{12^2 + 4^2 + 3^2}$$

$$= 13 \text{ m}$$

24. $\sqrt[5]{16} \times \sqrt[5]{2} \Rightarrow (16)^{1/5} \times (2)^{1/5}$

$$\Rightarrow (32)^{1/5}$$

$$\Rightarrow (2^5)^{1/5} \Rightarrow 2$$

25. If $x = \sqrt{2} - 1$ then $\frac{1}{x} = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$$\frac{1}{x} = (\sqrt{2} + 1)$$

$$\text{so value of } \left(\frac{1}{x} - x \right)^3 \Rightarrow [(\sqrt{2} + 1) - (\sqrt{2} - 1)]^3$$

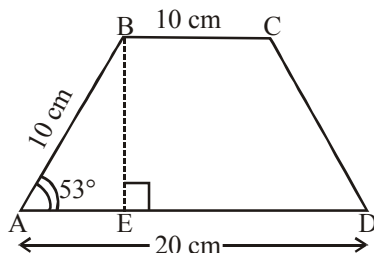
$$\Rightarrow 2^3 \Rightarrow 8$$

26. In $\triangle ABE$

$$\sin 53^\circ = \frac{BE}{AB}$$

$$\frac{4}{5} = \frac{BE}{10}$$

$$BE = 8 \text{ cm}$$



$$\text{Area of trapezium ABCD} = \frac{1}{2}(AD + BC) \times BE$$

$$= \frac{1}{2} \times 30 \times 8 \Rightarrow 120 \text{ cm}^2$$

27. $y - x = 80$ (1)

$y + x = 180$ (2)

Add (1) + (2)

$$y = 130, x = 50$$

30. Volume of sphere ($R = 8 \text{ cm}$)

$$= n \times \text{volume of sphere } (r = 2 \text{ cm})$$

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$R^3 = nr^3 \Rightarrow (8)^3 = n \times (2)^3$$

$$n = 4^3 \Rightarrow n = 64$$

31. $f(2) = a(2) + b$

$$2a + b = 1 \quad \dots(1)$$

$$f(-3) = a(-3) + b$$

$$-3a + b = 11 \quad \dots(2)$$

After solving equation (1) & (2)

$$a = -2, b = 5$$

$$\text{so } f(x) = -2x + 5$$

32. (a) $\sqrt{99} = (100 - 1)^{1/2}$

$$\Rightarrow 10 \left[1 - \frac{1}{100} \right]^{1/2}$$

$$\approx 10 \left[1 - \frac{1}{2} \times \frac{1}{100} \right]$$

$$\approx \frac{10 \times 199}{200}$$

$$\approx 9.95$$

(b) $\frac{1}{1.01} \Rightarrow (1.01)^{-1}$

$$\Rightarrow (1 + 0.01)^{-1} \approx (1 - 0.01)$$

$$\approx 0.99$$

33. In $\triangle ABO$ & $\triangle CBD$

$$\frac{OB}{AO} = \frac{DB}{CD}$$

$$\frac{4}{AO} = \frac{6}{12}$$

$$AO = 8$$

So coordinate of points A(0, -8)

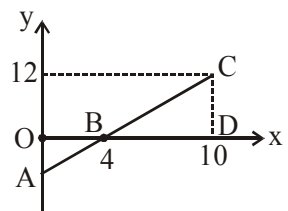
B(4, 0)

$$\text{slope of line } (m) = 2$$

So equation of line

$$y = mx + C$$

$$y = 2x - 8$$



VECTORS : RACE # 01

SOLUTION

12. $\vec{R} = \vec{a} + \vec{b} + \vec{c}$

$$= 4\hat{i} - \hat{j} - 3\hat{i} - \hat{k} = \hat{i} - \hat{j} - \hat{k}$$

$$\text{unit vector} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$$

13. $R = 2F \cos \theta/2 \Rightarrow 40\sqrt{3}$

$$2F \times \cos 30^\circ = 40\sqrt{3}$$

$$\Rightarrow F = 40 \text{ N}$$

14. $R = \sqrt{2^2 + 3^2 + 2 \times 2 \times 3 \cos \theta} \dots (1)$

$$2R = \sqrt{6^2 + 2^2 + 2 \times 6 \times 2 \cos \theta} \dots (2)$$

$$\frac{1}{4} = \frac{4 + 9 + 12 \cos \theta}{36 + 4 + 24 \cos \theta}$$

$$4(13 + 12 \cos \theta) = 40 + 24 \cos \theta$$

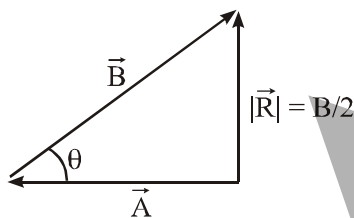
$$52 + 48 \cos \theta = 40 + 24 \cos \theta$$

$$24 \cos \theta = -12$$

$$\Rightarrow \cos \theta = -\frac{1}{2}$$

$$\Rightarrow \theta = 120^\circ$$

15.



$$\sin \theta = \frac{B/2}{B} = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\text{Angle between } \vec{A} \text{ \& } \vec{B} = 180^\circ - 30^\circ = 150^\circ$$

16. $|\vec{OA} + \vec{OB} + \vec{OC}| = |(\vec{OA} + \vec{OC}) + \vec{OB}|$

Resultant of $\vec{OA}$ and $\vec{OC}$ is along $\vec{OB}$ and

$$|\vec{OA} + \vec{OB} + \vec{OC}| = r\sqrt{2} + r = r(1 + \sqrt{2})$$

17. $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$$\sqrt{a^2 + b^2 + 2ab \cos \theta} = \sqrt{a^2 + b^2 - 2ab \cos \theta}$$

Square both the side

$$a^2 + b^2 + 2ab \cos \theta = a^2 + b^2 - 2ab \cos \theta$$

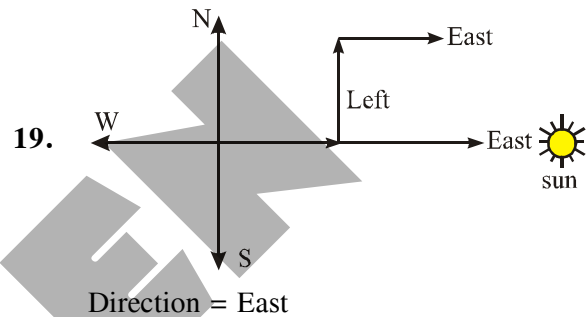
$$4ab \cos \theta = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

18. For (i) $8 \text{ cm} \leq \text{Displacement} \leq 16 \text{ cm}$

For (ii) $4 \text{ cm} \leq \text{Displacement} \leq 12 \text{ cm}$

For (iii) $2 \text{ cm} \leq \text{Displacement} \leq 14 \text{ cm}$

For (iv) $14 \text{ cm} \leq \text{Displacement} \leq 16 \text{ cm}$



19.

Direction = East

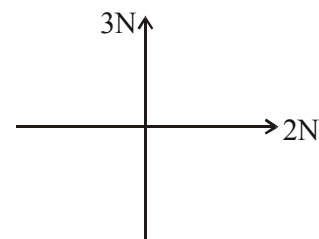
20. $\vec{A}, \vec{B}, \vec{C}$ are orthogonal vectors (it means angle between any of them is 90°)

$$\text{So Let } \vec{A} = 3\hat{i} \quad \vec{B} = 4\hat{j} \quad \vec{C} = 12\hat{k}$$

$$|\vec{A} - \vec{B} + \vec{C}| = |3\hat{i} - 4\hat{j} + 12\hat{k}|$$

$$= \sqrt{3^2 + 4^2 + 12^2} = 13$$

21.



for force along x-axis

$$F_{\min} = 3\text{N along } (-y \text{ axis})$$

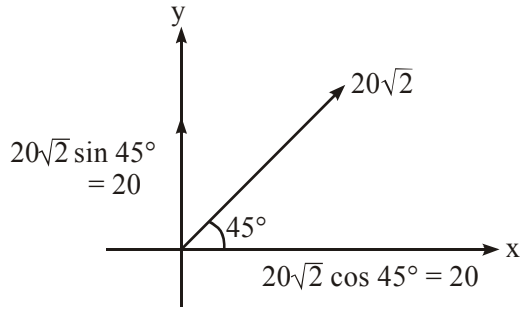
22. $\vec{a} + \vec{b} = 2\hat{i} \dots (1)$

$$\vec{b} = 3\hat{j} - \hat{k} \dots (2)$$

$$\text{eq. (1) - (2)}$$

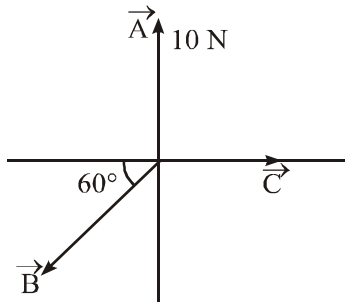
$$\therefore \Rightarrow \vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$$

2.



$$\vec{F} = 20\hat{i} + 20\hat{j}$$

3.



$$B \sin 60^\circ = 10$$

$$B = \frac{10 \times 2}{\sqrt{3}} = \frac{20}{\sqrt{3}}$$

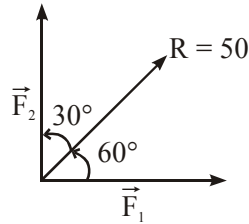
$$B \cos 60^\circ = C$$

$$\frac{20}{\sqrt{3}} \times \frac{1}{2} = C \Rightarrow C = \frac{10}{\sqrt{3}}$$

4. Length in x-y plane

$$= |3\hat{i} + \hat{j}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

5.



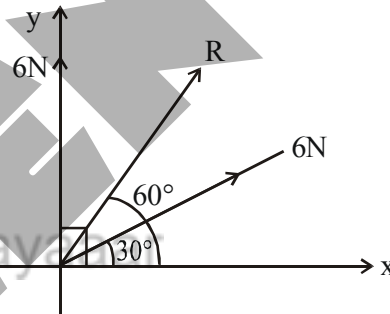
$$R \cos 60^\circ = F_1$$

$$F_1 = 50 \times \frac{1}{2} = 25$$

$$R \cos 30^\circ = F_2$$

$$F_2 = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3}$$

6.



$$F_{\text{net}} = \sqrt{6^2 + 6^2 + 2 \times 6 \times 6 \cos 60^\circ}$$

$$= \sqrt{36 + 36 + 2 \times 6 \times 6 \times \frac{1}{2}}$$

$$= \sqrt{36 + 36 + 36} = 6\sqrt{3} \text{ N}$$

$$= \sqrt{108} = 10.392 \text{ N}$$

VECTORS : RACE # 03

SOLUTION

- $\vec{P} = K\vec{Q}$
 $K = +ve \quad \vec{P} \parallel \vec{Q}$
 $K = -ve \quad \vec{P} \nparallel \vec{Q}$
- $\vec{A} \cdot \vec{B} = 3 \times 5 \times \cos \theta$
 $-15 \leq \vec{A} \cdot \vec{B} \leq 15$
- Projection of $\vec{A}$ on $\vec{B} = \frac{\vec{A} \cdot \vec{B}}{B}$

$$= \frac{\vec{A} \cdot \vec{B}}{B} = \frac{-2+9-4}{\sqrt{1^2+3^2+4^2}} \Rightarrow \frac{3}{\sqrt{26}}$$
- $|\hat{A} \times \hat{B}| = -\sqrt{3} \hat{A} \cdot \hat{B}$
 $\sin \theta = -\sqrt{3} \cos \theta \Rightarrow \tan \theta = -\sqrt{3} \Rightarrow \theta = 120^\circ$
 $|\hat{A} - \hat{B}| = \sqrt{1^2 + 1^2 - 2 \times 1 \times 1 \times \left(-\frac{1}{2}\right)} = \sqrt{3}$
- $$\vec{r} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ 2 & 1 & 4 \end{vmatrix}$$

$$= \hat{i}(12-1) - \hat{j}(28-2) + \hat{k}(7-6)$$

$$= 11\hat{i} - 26\hat{j} + \hat{k}$$
- $$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$$= \hat{i}(-24+6) - \hat{j}(18-5) + \hat{k}(-18+20)$$

$$= -18\hat{i} - 13\hat{j} + 2\hat{k}$$
- $\tan \alpha = \frac{A_y}{A_x} = \frac{1}{1} = 1 \Rightarrow \alpha = 45^\circ$
- $\vec{A} \perp \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = 0$
 $(3\hat{i} + 2\hat{j} + \hat{k}) \cdot (5\hat{i} - 9\hat{j} + P\hat{k}) = 0$
 $= 15 - 18 + P = 0 \Rightarrow P = 3$
- Area of $\Delta = \frac{1}{2} |\vec{A} \times \vec{B}|$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 3 & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2} (-\hat{j}(-3) + \hat{k}(-3)) = \frac{|3\hat{j} - 3\hat{k}|}{2}$$

$$= \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}}$$
- $\vec{F}_1 \cdot \vec{F}_2 = 2 \times 0 + 0 \times 3 + 5 \times 4 = 20$

- $\vec{A} \cdot \vec{B} = -AB$
 $AB \cos \theta = -AB$
 $\Rightarrow \cos \theta = -1$
 $\Rightarrow \theta = 180^\circ$ So $\vec{A} \parallel \vec{B}$
- $(\vec{A} + 2\vec{B}) \cdot (2\vec{A} - 3\vec{B})$
 $= 2A^2 - 3\vec{A} \cdot \vec{B} + 4\vec{B} \cdot \vec{A} - 6B^2$
 $= 2A^2 + AB \cos \theta - 6B^2$
- $\vec{A} = 3\hat{i} + 4\hat{j}, \vec{B} = 6\hat{i} + 8\hat{j}$
 $\vec{A} \cdot \vec{B} = 18 + 32 = 50$

Here $\hat{A} = \hat{B} = \frac{1}{5}(3\hat{i} + 4\hat{j})$

and $2\vec{A} = \vec{B}$ so $\vec{A} \times \vec{B} = \vec{0}$
- $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$
 $\theta = 60^\circ$
- Unit vector perpendicular to both $\vec{A}$ & $\vec{B}$ is

$$\frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\therefore \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = (\hat{k}(-2-1)) = -3\hat{k}$$

$$\Rightarrow \frac{-3\hat{k}}{3} \Rightarrow -\hat{k} \text{ or } \hat{k}$$
- Projection of velocity along $\vec{a}$

$$\left(\frac{\vec{v} \cdot \vec{a}}{a}\right) \left(\frac{\vec{a}}{a}\right)$$

$$\Rightarrow \left(\frac{6+2-2}{\sqrt{3}}\right) \left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right) = \frac{6}{3}(\hat{i}+\hat{j}+\hat{k})$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k}$$
- $\vec{A} = \lambda \vec{B}$
 $= 4\hat{i} + \alpha\hat{j} + 2\hat{k} = \lambda[2\hat{i} + \hat{j} + \hat{k}]$

Compare $\lambda = 2$ & $\alpha = 2$
- $\vec{AB} = (4\hat{i} + 5\hat{j} + 6\hat{k}) - (3\hat{i} + 4\hat{j} + 5\hat{k})$
 $= \hat{i} + \hat{j} + \hat{k}$
 $\vec{CD} = (4\hat{i} + 6\hat{j}) - (7\hat{i} + 9\hat{j} + 3\hat{k})$
 $= -3\hat{i} - 3\hat{j} - 3\hat{k} = -3(\hat{i} + \hat{j} + \hat{k})$

So $\vec{AB} \parallel \vec{CD}$

UNITS, DIMENSIONS & MEASUREMENT : RACE # 01

SOLUTIONS

9. $n_1 u_1 = n_2 u_2$

$$1 \times \left(\frac{\text{kg} \times \text{m}^2}{\text{s}^2} \right) = n_2 \left(\frac{10 \text{kg} \times (10 \text{m})^2}{(60 \text{s})^2} \right)$$

$$n_2 = \frac{60 \times 60}{10 \times 100} = 3.6$$

10. $[B] = [x^2] = [L^2]$

$$[U] = \frac{[A] \left[L^{\frac{1}{2}} \right]}{[L^2]} = [M^1 L^2 T^{-2}]$$

$$[A] = [M^1 L^{\frac{7}{2}} T^{-2}]$$

$$[AB] = [M^1 L^{\frac{7}{2}} T^{-2}] [L^2] = [M^1 L^{\frac{11}{2}} T^{-2}]$$

13. $[E] = [\text{work}] = [F] \times [L]$

$$= \left[\frac{P}{t} \right] \times \left[A^{\frac{1}{2}} \right]$$

$$= [P^1 A^{\frac{1}{2}} T^{-1}]$$

14. For accuracy we can look for percentage error

$$\text{For (i), (2), (4) \% error} = \frac{0.1}{63.1} \times 100\%$$

$$\text{For (3) \% error} = \frac{1}{6310} \times 100\%$$

15. $a + b = 14 \pm 0.14 \Rightarrow \% \text{ error} = \frac{0.14}{14} \times 100 = 1\%$

$$a - b = 2 \pm 0.14 \Rightarrow \% \text{ error} = \frac{0.14}{2} \times 100 = 7\%$$

$$a \times b = 48 \pm 0.96 \Rightarrow \% \text{ error} = \frac{0.96}{48} \times 100 = 2\%$$

so, order of \% error $x < z < y$

17. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{dR_{eq}}{R^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2} \quad \therefore \% \text{ error} = \frac{4}{3}\%$$

18. $\frac{dm}{m} = \left(\frac{\sec^2 \theta}{\tan \theta} \right) d\theta = \frac{d\theta}{2(\sin 2\theta)}$

20. Perimeter $p = 2(\ell + b) = 2(10.5 + 5.2) = 31.4 \text{ cm}$
 $\Delta p = 2(\Delta \ell + \Delta b) = 2(0.2 + 0.1) = 0.6 \text{ cm}$

21. $\frac{\Delta Y}{Y} \times 100 = \left(\frac{\Delta W}{W} + \frac{\Delta L}{L} + \frac{2\Delta r}{r} + \frac{\Delta \ell}{\ell} \right) \times 100$
 $= 0.5\% + 1\% + 2 \times 3\% + 4\% = 11.5\%$

22. Precision for a set of measurement is seen by least count of instrument.

UNITS, DIMENSIONS & MEASUREMENT : RACE # 02

SOLUTIONS

1. Reading = 2.30 mm
2. Least count = 1 MSD - 1 VSD

$$(N-1) \text{ MSD} = N(\text{VSD}) \Rightarrow \text{VSD} = \left(1 - \frac{1}{N}\right) \text{MSD}$$

$$\text{Least count} = 1\text{MSD} - \left(1 - \frac{1}{N}\right) \text{MSD}$$

$$= \left(\frac{1}{N}\right) (1\text{mm}) = \frac{1}{10N} \text{ cm}$$
3. $\therefore \text{L.C.} = 1 \text{ MSD} - 1 \text{ VSD}$
 $\therefore 0.02 = 0.1 - \frac{m}{n} \Rightarrow \frac{m}{n} = 0.08$
4. Least count = $\frac{1\text{mm}}{100} = 0.01 \text{ mm}$
 Diameter of the wire = $0 + 52 \times 0.01 \text{ mm}$
 $= 0.52 \text{ mm} = 0.052 \text{ cm}$
5. least count = $\frac{0.5}{50} = 0.01 \text{ mm}$
 Diameter of sphere
 $= 2 \times 0.5 + (25 - 5) \times 0.01 = 1.2 \text{ mm}$
6. 50 divisions = 2.45 cm
 $\Rightarrow 1 \text{ division} = \frac{2.45}{50} = 0.049 \text{ cm}$
 $\Rightarrow \text{least count} = 1 \text{ MSD} - 1\text{VSD} = 0.05 - 0.049$
 $= 0.001 \text{ cm.}$
 So vernier reading = $0.001 \times 24 = 0.024 \text{ cm.}$
 Therefore diameter of cylinder
 $= 5.10 + 0.024 = 5.124 \text{ cm.}$
7. Diameter = M.S.R. + C.S.R $\times$ L.C. + Z.E.
 $= 3 + 35 \times (0.5/50) + 0.03$
 $= 3.38 \text{ mm}$

8. Least count = $\frac{\text{pitch}}{\text{No. of divisions}} = \frac{2}{200} = 0.01 \text{ mm};$
 Reading = $3 \times 2 + (46-5) (0.01) = 6.41 \text{ mm}$
9. Least count = 1MSD - 1VSD
 but $30 \text{ VSD} = 29 \text{ MSD} \Rightarrow \text{VSD} = \left(\frac{29}{30}\right) \text{MSD}$
 Therefore L.C. = $1 \text{ MSD} - \frac{29}{30} \text{MSD}$

$$= \frac{\text{MSD}}{30} = \frac{0.5^\circ}{30} = \left(\frac{1}{60}\right)^\circ = 1'$$
12. Zero error (excess reading) = 0.3 mm.
 observed thickness of block = 13.7 mm.
 Actual thickness = $13.7 - 0.3 = 13.4 \text{ mm.}$
13. Least count = $\frac{1\text{mm}}{10} = 0.1 \text{ mm};$
 Zero error = $-(10-6) \times 0.1 = -0.4 \text{ mm}$
 Reading = $6 + 5 \times (0.1) - (-0.4) = 6.9 \text{ mm}$
14. Least count = $\frac{1\text{mm}}{100} = 0.01 \text{ mm}$
 Diameter $D = 1\text{mm} + (47) (0.01)\text{mm} = 1.47\text{mm}$
 $= 0.147 \text{ cm}$
 Curved surface area = $2\pi r\ell = \pi D\ell$
 $= (3.14) (0.147) (5.6) = 2.6 \text{ cm}^2$
15. (a) Precise = Yes
 Accurate = No
 (b) Accurate = Yes
 Precise = Yes
 (c) Accurate = Yes
 Precise = No

KINEMATICS : RACE # 01

SOLUTION

1. $\vec{s}_1 = -25\hat{i}$

$\vec{s}_2 = 10\hat{i} - 10\sqrt{3}\hat{j}$

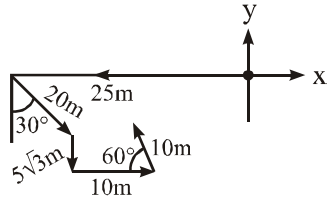
$\vec{s}_3 = -5\sqrt{3}\hat{j}$

$\vec{s}_4 = 10\hat{i}$

$\vec{s}_5 = -5\hat{i} + 5\sqrt{3}\hat{j}$

$\Sigma \vec{s}_i = -10\hat{i} - 10\sqrt{3}\hat{j}$

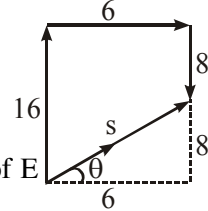
$|\vec{s}| = 20\text{m}$ & $\tan \theta = \sqrt{3}$ $\theta = 60^\circ$ south of west



2. Distance = $16 + 6 + 8 = 30$

displacement = $\sqrt{6^2 + 8^2} = 10$

$\tan \theta = \frac{8}{6} = \frac{4}{3} \therefore \theta = 53^\circ$, N of E



time = $\frac{30}{15} = 2$ sec.

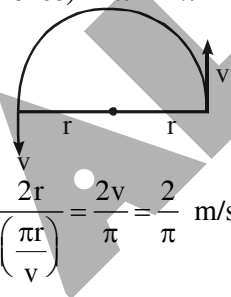
$\langle \vec{v} \rangle = \frac{\text{displacement}}{\text{time}} = \frac{10}{2} = 5 \text{ m/s}$, 53° , N of E

3. Displacement $s = 2r = 2\text{m}$

Distance $d = \frac{1}{2}$ (circumference) = $\pi r = \pi\text{m}$

time = $\frac{\text{distance}}{\text{speed}} = \frac{\pi}{v}$

Avg. velocity $\langle v \rangle = \frac{s}{t} = \frac{2r}{\left(\frac{\pi r}{v}\right)} = \frac{2v}{\pi} = \frac{2}{\pi} \text{ m/s}$



Avg. acceleration $\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{-v\hat{j} - (+v\hat{j})}{\left(\frac{\pi r}{v}\right)} = \frac{-2v^2\hat{j}}{\pi r} \text{ m/s}^2$

$= \frac{-2}{\pi} \hat{j} \text{ m/s}^2$ magnitude = $\frac{2}{\pi} \text{ m/s}^2$

4. $\langle v \rangle = \frac{d_1 + d_2}{t_1 + t_2} = \frac{200,000 + 50,000}{\frac{200,000}{40} + \frac{50,000}{10}}$

$= \frac{250}{10} = 25 \text{ m/s}$

5. $\langle v \rangle = \frac{3000 + 3000 + 3000}{\frac{3000}{2} + \frac{3000}{3} + \frac{3000}{6}} = \frac{9}{3} = 3 \text{ m/s}$

6. $\langle v \rangle = \frac{[30 \times 3] + 90}{3 + \frac{90}{45}} = \frac{180}{5} = 36 \text{ km/hr}$

7. $\langle v_A \rangle = \frac{2v_1v_2}{v_1 + v_2} = 48 \text{ km/hr}$

$\langle v_B \rangle = \frac{v_1 + v_2}{2} = 50 \text{ km/hr}$

$(v_{\text{avg}})_B > (v_{\text{avg}})_A \therefore t_B < t_A$

$t_A = \frac{240}{48} = 5 \text{ hour}$, $t_B = \frac{240}{50} = 4.8 \text{ hour}$

$\Delta t = [t_A - t_B] = 0.2 \text{ hr} = 12 \text{ minutes before}$

8. (i) $s = 2r \sin \theta/2$

$s = 2r \sin \left(\frac{60^\circ}{2}\right) = 2r \times \frac{1}{2} = r$

(ii) $d = \text{arc} = \theta r$

$d = \frac{\pi}{3} r$

time = $\frac{\text{dist.}}{\text{speed}} = \left[\frac{\theta r}{v}\right]$

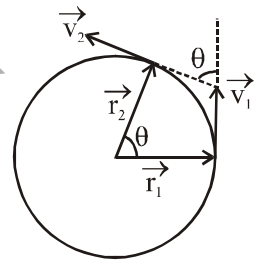
$t = \frac{\pi r}{3v}$

$\langle v \rangle = \frac{s}{t} = \frac{r}{\left(\frac{\pi r}{3v}\right)} = \left(\frac{3v}{\pi}\right)$

$\left[\because \Delta \vec{v} = \vec{v}_2 - \vec{v}_1, |\Delta \vec{v}| = 2v \sin \left(\frac{\theta}{2}\right)\right]$

$|\langle \vec{a} \rangle| = \left|\frac{\Delta \vec{v}}{\Delta t}\right| = \frac{2v \sin \frac{\theta}{2}}{t}$

$= \frac{2v \sin \frac{60^\circ}{2}}{t} = \frac{v}{t} = \frac{v}{\left(\frac{\pi r}{3v}\right)} = \frac{3v^2}{\pi r}$



9. $\langle v \rangle = \frac{x}{\frac{(x/3)}{v_1} + \frac{(2x/3)}{v_2}} = \frac{3v_1v_2}{v_2 + 2v_1}$

10.

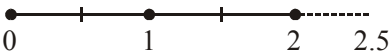


$\langle v_0 \rangle = \frac{2v_2v_3}{v_2 + v_3} = 4 \text{ m/s}$

$\langle v \rangle = \frac{v_1 + \langle v_0 \rangle}{2} = 4.5 \text{ m/s}$

KINEMATICS : RACE # 02

SOLUTION

1. 

$$S_{5\text{th half}} = S_{2.5} - S_2$$

$$= \frac{1}{2} 2[(2.5)^2 - 2^2] = 2.25 \text{ m}$$

2. $u = 10 \text{ m/s}$

$$t = 5 \text{ s}$$

$$v = 0$$

$$s = ?$$

$$(i) s = \left(\frac{v+u}{2} \right) t$$

$$s = \left[\frac{10+0}{2} \right] 5 = 25 \text{ m}$$

(ii) No

$$(iii) \text{ Retardation} = \frac{u-v}{t} = \frac{10}{5} = 2 \text{ ms}^{-2}$$

$$(iv) F = m|\vec{a}| = 1000 \times |-2| = 2000 \text{ N}$$

3. $2u + \frac{1}{2} a(2)^2 = 14 \Rightarrow u + a = 7 \dots (1)$

$$\& 6u + \frac{1}{2} a(6)^2 = 88 + 14 = 102$$

$$u + 3a = 17 \dots (2)$$

from equation (1) & (2)

$$2a = 10$$

$$a = 5$$

$$u = 2$$

$$S_{5\text{th}} = 2 + \frac{5}{2} (2 \times 5 - 1)$$

$$= 24.5 \text{ m}$$

4. $u = 0 \quad v = v \quad t = n$

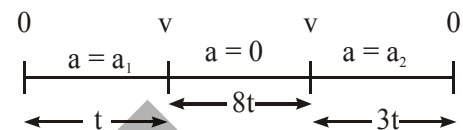
$$a = \left(\frac{v}{n} \right)$$

distance covered in last 't' seconds.

$$S = vt - \frac{1}{2} at^2$$

$$= v(2) - \frac{1}{2} \times \frac{v}{n} \times (2)^2$$

$$= 2v - \frac{2v}{n} = 2v \left[1 - \frac{1}{n} \right]$$

5. 

$$a_1 = \frac{v}{t} \& a_2 = \frac{v}{3t} \text{ [by I equation of motion]}$$

(since distance is always positive)

$$d = \frac{v^2}{2a_1} + v(8t) + \frac{v^2}{|2a_2|}$$

$$= \frac{vt}{2} + 8vt + \frac{3vt}{2}$$

$$d = 10 vt$$

$$\langle v \rangle = \frac{10vt}{12t} = \frac{5}{6} \times 60 = 50 \text{ km/hr}$$

6. $s_1 = \frac{1}{2} at^2 = \frac{1}{2} \times 2 \times 100 = 100$

$$v = at = 20 \text{ m/s}$$

$$s_2 = vt = 20 \times 30 = 600 \text{ m}$$

$$s_3 = \frac{v^2}{2a} = \frac{400}{2 \times 4} = 50$$

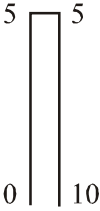
$$\Sigma s = 750 \text{ m}$$

KINEMATICS : RACE # 03

SOLUTION

1. Distance covered in last second of upward journey = distance covered in 1st sec. of downward journey.

$$d = \frac{1}{2} g [1]^2 = 4.9 \text{ m}$$

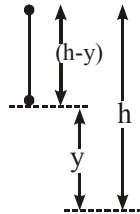
2.  $S_{1^{\text{st}}} = S_{10^{\text{th}}}$
 $S_{2^{\text{nd}}} = S_{9^{\text{th}}}$
 $S_{3^{\text{rd}}} = S_{8^{\text{th}}}$
 $S_{4^{\text{th}}} = S_{7^{\text{th}}}$
 $S_{5^{\text{th}}} = S_{6^{\text{th}}}$

so Answer will be Option (1)

3. $v^2 = 2as$

$$v^2 = 2g(h - y)$$

$$v = \sqrt{2g(h - y)}$$



4. $v = u + at$

$$v = 6 - (10)\left(\frac{1}{2}\right)$$

$$v = 1 \text{ m/s}$$

5. $\frac{v_0^2}{2g} = h$

$$(i) v_0 = \sqrt{2 \times 10 \times 180} = 60 \text{ m/s}$$

$$(ii) t = \frac{v_0}{g} = 6 \text{ s}$$

$$(iii) T = 2t = 12 \text{ s}$$

$$(iv) h = (60) 8 - \frac{1}{2} g (8)^2 = 160 \text{ m}$$

$$(v) \vec{v} = \vec{u} + \vec{a}t = 60 - (10)(8)$$

$$= -20 \text{ m/s}$$

$$= 20 \text{ m/s downward}$$

$$(vi) 100 = 60t - \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow \text{After solving } t = 2 \text{ sec or } 10 \text{ sec}$$

- (vii) We have to find time to travel O to A.

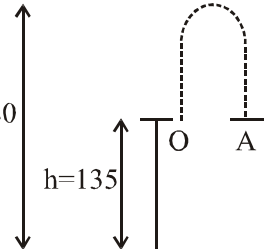
Let t be the time between O and A.

from top-most point distance travelled

$$= (180 - 135) = 45 \text{ m}$$

$$\therefore t_1 = \sqrt{\frac{2 \times 45}{g}} = 3 \text{ sec}$$

$$t = 2t_1 = 6 \text{ sec}$$

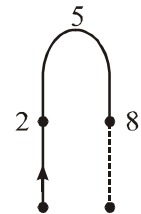


6. (i) $t = \frac{2+8}{2} = 5 \text{ s}$

$$T = 2t = 10 \text{ sec}$$

$$(ii) u = gt = 50 \text{ m/s}$$

$$(iii) h = \frac{u^2}{2g} = 125 \text{ m}$$



$$(iv) h(2, 8) = 50 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 80 \text{ m}$$

7. $\frac{1}{2} g(t+2)^2 - \frac{1}{2} gt^2 = 80 \text{ m}$

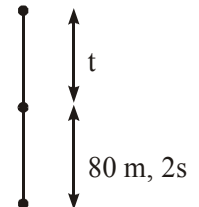
$$t^2 + 4 + 4t - t^2 = 16$$

$$\Rightarrow t = 3 \text{ sec}$$

$$(i) h = \frac{1}{2} g(3+2)^2 = 125 \text{ m}$$

$$(ii) t = 3 + 2 = 5 \text{ sec}$$

$$(iii) v = g(t+2) = 50 \text{ m/s}$$



8. Time first drop has fallen by $= \sqrt{\frac{2H}{g}} = 4 \text{ gap}$

$$\text{gap} = \frac{1}{4} \sqrt{\frac{2H}{g}}$$

$$\text{Time of falling for 4th drop} = 1 \text{ gap} = \frac{1}{4} \sqrt{\frac{2H}{g}}$$

$$\text{for 3rd drop} = 2 \text{ gap} = \frac{1}{2} \sqrt{\frac{2H}{g}}$$

$$\text{for 2nd drop} = 3 \text{ gap} = \frac{3}{4} \sqrt{\frac{2H}{g}}$$

$$(i) \text{ Height of 2nd drop} = H - \frac{1}{2}gt^2$$

$$= H - \frac{1}{2}g \left[\frac{9}{16} \times \frac{2H}{g} \right] = \frac{7}{16}H$$

$$(ii) = d_{23} = \frac{1}{2}g \left[\frac{3}{4} \sqrt{\frac{2H}{g}} \right]^2 - \frac{1}{2}g \left[\frac{1}{2} \sqrt{\frac{2H}{g}} \right]^2$$

$$= \frac{5}{16}H$$

$$(iii) v = gt = g \frac{1}{2} \sqrt{\frac{2H}{g}} = \sqrt{\frac{gH}{2}}$$

OR

According to Galileo's Law ratio of distances in equal interval = 1 : 3 : 5 : 7 :

$$16x = H$$

$$x = \frac{H}{16}$$

$$(i) \frac{7H}{16}$$

$$(ii) \frac{5H}{16}$$

$$(iii) v = \sqrt{2g \frac{4H}{16}}$$

$$v = \sqrt{\frac{gH}{2}}$$



KINEMATICS : RACE # 04

SOLUTION

$$1. \quad \frac{u^2 \sin 2\theta}{g} = n \left[\frac{u^2 \sin^2 \theta}{2g} \right]$$

$$\Rightarrow 2 \sin \theta \cos \theta = \frac{n \sin^2 \theta}{2} \Rightarrow \frac{4}{n} = \tan \theta$$

$$\Rightarrow \frac{PE}{KE} = \frac{mgH}{\frac{1}{2}mv^2} = \frac{2gH}{v^2} = \frac{[2g]}{u^2 \cos^2 \theta} \left[\frac{u^2 \sin^2 \theta}{2g} \right]$$

$$= \tan^2 \theta \Rightarrow \frac{16}{n^2}$$

$$2. \quad T = \frac{2u \sin \theta}{g} = 10 \quad \dots(i)$$

$$R = \frac{u^2 \sin 2\theta}{g} = 500 \quad \dots(ii)$$

Squaring (i) & then dividing by (ii)

$$\Rightarrow \frac{4 \sin^2 \theta}{g \sin 2\theta} = \frac{1}{5}$$

$$\frac{4 \sin^2 \theta}{10 \times 2 \sin \theta \cos \theta} = \frac{1}{5}$$

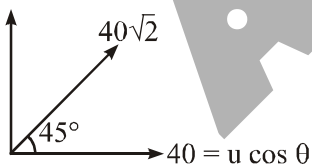
$$\tan \theta = 1$$

$$\theta = 45^\circ$$

from (i) $u = 50\sqrt{2}$ m/s

$\vec{v} = (u \sin \theta - gt)\hat{j} + u \cos \theta \hat{i}$ and at $t = 5$ sec

$|\vec{v}| = 50$ m/s or $\vec{v} = 50\hat{i}$ m/s

$$3. \quad u \sin \theta = 40$$


$$(i) \quad H_{\max} = \frac{u^2 \sin^2 \theta}{2g} = 80 \text{ m}$$

$$(ii) \quad T = \frac{2u \sin \theta}{g} = 8 \text{ s}$$

$$(iii) \quad R = u_x \times T$$

$$R = 40 \times 8 = 320 \text{ m}$$

$$(iv) \quad \text{velocity at } H_{\max} = u \cos \theta = 40 \text{ m/s}$$

$$(v) \quad \frac{KE}{PE} = \frac{v^2}{2gH} = \frac{u^2 \cos^2 \theta}{2g \left[\frac{u^2 \sin^2 \theta}{2g} \right]} = \cot^2 \theta = 1$$

$$(vi) \quad y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$y = x - \frac{10x^2}{2 \times 40 \times 40}$$

$$y = x - \frac{x^2}{320}$$

$$(vii) \quad x = u_x \times T$$

$$40 \times 2 = 80 \text{ m}$$

$$(viii) \quad y = (u \sin \theta) t - \frac{1}{2}gt^2 = 60 \text{ m}$$

$$4. \quad \frac{u^2 2 \sin \theta \cos \theta}{g} = 4 \left(\frac{u^2 \sin^2 \theta}{2g} \right)$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$

$$R = \frac{u^2 \sin(2 \times 45^\circ)}{g} = \frac{u^2}{g}$$

$$H_{\max} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g}$$

At maximum height

$$\frac{KE}{PE} = \frac{\frac{1}{2}m[u \cos 45^\circ]^2}{mgH_{\max}} = \frac{1}{1}$$

5. From previous question observation here R is 4 times of H.

So $\theta = 45^\circ$

$$R = 20 \text{ m}$$

$$\Rightarrow \frac{u^2}{g} \sin 2 \times 45^\circ = 20$$

$$u^2 = 20g \Rightarrow u = 10\sqrt{2} \text{ ms}^{-1}$$

$$T = \frac{2u \sin 45^\circ}{g} = \frac{2 \times 10\sqrt{2} \times \frac{1}{\sqrt{2}}}{10} = 2 \text{ sec}$$

6.

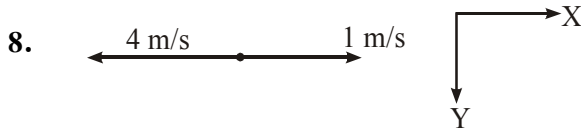


(i) Yes

(ii) It would be straight line as relative acceleration is zero

(iii) Will miss by $1/2 gt^2$.

7. (i) As their vertical motion is same they will reach at same time.
(ii) Vertical velocities will be same as their vertical motion is same



$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{v}_1 = (\hat{i}) - (10\hat{j})t$$

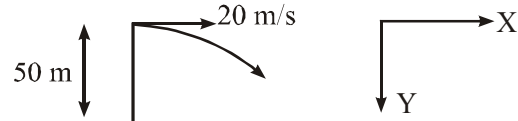
$$\vec{v}_2 = -4\hat{i} - (10\hat{j})t$$

$$\vec{v}_1 \perp \vec{v}_2$$

$$\begin{aligned} \text{so } \vec{v}_1 \cdot \vec{v}_2 &= 0 \\ -4 + 100t^2 &= 0 \\ t &= 0.2 \text{ s} \end{aligned}$$

9. $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$
From given initial velocity
 $u = \sqrt{u_x^2 + u_y^2}$
 $u = \sqrt{1^2 + 2^2} = \sqrt{5}$
 $\tan \theta = \frac{u_y}{u_x}$
 $\tan \theta = 2$
so $y = 2x - \frac{5x^2}{1} = 2x - 5x^2$

10.



$$(i) \vec{v} = (gt)\hat{j} + (u_x)\hat{i} = 20\hat{j} + 20\hat{i}$$

$$\Rightarrow |\vec{v}| = 20\sqrt{2} \text{ m/s}$$

$$(ii) x = u_x t$$

$$x = 20 \times 3 = 60 \text{ m}$$

$$y = \frac{1}{2}gt^2$$

$$y = \frac{1}{2} \times 10 (3)^2 = 45 \text{ m}$$

$$\sqrt{x^2 + y^2} = \sqrt{45^2 + 60^2} = 15 \times 5 = 75 \text{ m away.}$$

(iii) Time taken to reach the ground

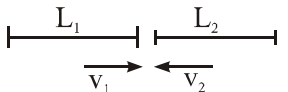
$$= \sqrt{\frac{2H}{g}} = \sqrt{\frac{2(50)}{10}} = \sqrt{10} \text{ s}$$

$$\begin{aligned} \text{velocity} &= \sqrt{u_x^2 + (gT)^2} \\ &= \sqrt{(20)^2 + (10\sqrt{10})^2} \\ &= \sqrt{400 + 1000} \\ &= \sqrt{1400} = 37.4 \text{ m/s} \end{aligned}$$

$$(iv) |\Delta \vec{v}| = |\vec{a}t| = 20 \text{ m/s}$$

KINEMATICS : RACE # 05

SOLUTION

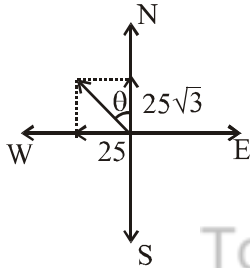
1. (i) $t = \frac{d_{rel}}{v_{rel}} = \frac{L_1 + L_2}{v_1 + v_2}$ 
(ii) $d_1 = \left[\frac{L_1 + L_2}{v_1 + v_2} \right] v_1$, $d_2 = \left(\frac{L_1 + L_2}{v_1 + v_2} \right) v_2$ (by $d = v \times t$)

2. (i) $t = \frac{d_{rel}}{v_{rel}} = \frac{d}{v_p - v_t} = \frac{150}{5} = 30 \text{ sec}$

(ii) $d_p = v_p t = 300 \text{ m}$

3. $t = \frac{\ell_b + \ell_t}{V_t} = \frac{150 + 100}{10} = 25 \text{ s}$

4. $\vec{v}_{bc} = \vec{v}_b - \vec{v}_c = 25\sqrt{3}\hat{j} - 25\hat{i}$
 $|\vec{v}_{bc}| = 50 \text{ km/hr}$



$\tan \theta = \frac{25}{25\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$

direction 30° W of N

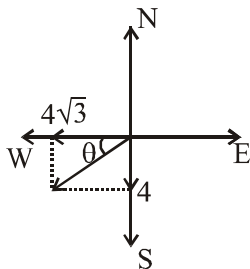
5. From previous formula

$25\sqrt{3}\hat{j} = \vec{v}_b - 25\hat{i}$

$\vec{v}_b = 25\sqrt{3}\hat{j} + 25\hat{i} \Rightarrow |\vec{v}_b| = 50 \text{ km/hr}$

$\tan \theta = \frac{25\sqrt{3}}{25} = 60^\circ$ North of East

6. $\vec{v}_{wo} = \vec{v}_w - \vec{v}_o = (-4\hat{j}) - (4\sqrt{3}\hat{i})$
 $= -4\sqrt{3}\hat{i} - 4\hat{j} \Rightarrow |\vec{v}_{wo}| = 8 \text{ km/hr}$



$\tan \theta = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$ $\vec{v}_{wo} = 10 \text{ km/hr}$ 30° S of W

7. $\vec{v}_{wo} = \vec{v}_w - \vec{v}_o \Rightarrow 5\sqrt{3}\hat{j} = \vec{v}_w - (-5\hat{i})$

$\Rightarrow \vec{v}_w = 5\sqrt{3}\hat{j} - 5\hat{i}$

$|\vec{v}_w| = 10 \text{ km/hr}$

$\tan \theta = \frac{5\sqrt{3}}{5} = \sqrt{3}$

$\theta = 60^\circ$ North of West or 30° West of North

8. $\vec{v}_o = 2\hat{i}$

$\vec{v}_{wo} = 2\hat{j}$

$\Rightarrow \vec{v}_{wo} = \vec{v}_w - \vec{v}_o$

$\vec{v}_w = 2\hat{i} + 2\hat{j}$

Now $\vec{v}_o = 4\hat{i}$

$\vec{v}_w = 2\hat{i} + 2\hat{j}$

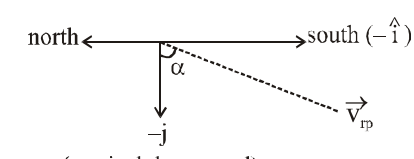
$\vec{v}_{wo} = 2\hat{i} + 2\hat{j} - 4\hat{i}$

$= -2\hat{i} + 2\hat{j}$

$|\vec{v}_{wo}| = 2\sqrt{2} \text{ m/s}$

$\tan \theta = \frac{2}{2} = 1$

$\theta = 45^\circ$, so direction is North-West

9. $\vec{v}_r = -2\hat{j}$ 

$\vec{v}_p = 2\sqrt{3}\hat{i}$

(vertical downward)

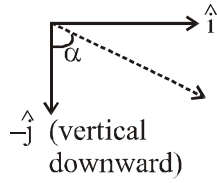
$\vec{v}_{rp} = -2\hat{j} - 2\sqrt{3}\hat{i}$

$|\vec{v}_{rp}| = \sqrt{4 + 12} = 4 \text{ m/s}$

$\tan \alpha = \left(\frac{2\sqrt{3}}{2} \right)$

$\alpha = 60^\circ$ with vertical

10.



$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

$$\vec{v}_r = \vec{v}_{rm} + \vec{v}_m \Rightarrow \vec{v}_r = 2\hat{i} - 2\sqrt{3}\hat{j}$$

$$|\vec{v}_r| = 4 \text{ km/hr} \quad \tan \alpha = \left(\frac{2}{2\sqrt{3}} \right)$$

$\alpha = 30^\circ$ from vertical

11. $V_B \cos \theta = V_A \Rightarrow \frac{V_A}{V_B} = \cos \theta$

[Relative velocity in horizontal direction should be zero]

NEWTON'S LAWS OF MOTION : RACE # 01

SOLUTION

1. Inertia of body depends on mass. not on velocity.

Hence option (3)

2. Change in momentum = Impulse

$$= \int F dt = \int_1^3 (12t + 3t^2) dt$$

$$= [6t^2 + t^3]_1^3 = 74 \text{ kg ms}^{-1}$$

3. $F = \frac{dP}{dt}$ = Slope of P-t curve

So $\frac{F_A}{F_B} = \frac{\tan 30^\circ}{\tan 45^\circ} = \frac{1}{\sqrt{3}}$

4. Impulse = $\int F dt$ = Area under F - t curve

$$= \frac{1}{2} (2 \times 100) - \frac{1}{2} \times 1(100) = 50 \text{ N-s.}$$

5. Impulse = $\Delta P \Rightarrow \vec{F} = \frac{\Delta \vec{P}}{\Delta t} = \frac{m(\vec{v}_2 - \vec{v}_1)}{t_2 - t_1}$

$$F = \frac{2(20 - \{-20\})}{0.1} = 800 \text{ N}$$

6. $F = nmv$ where n is no. of bullets per sec.

$$400 = n (40 \times 10^{-3}) (10^3)$$

$$\Rightarrow n = 10 \text{ bullets per sec.}$$

$$\text{Bullets per minute} = 10 \times 60 = 600$$

7. $F = m_1(4) = m_2(6) \Rightarrow m_1 = \frac{F}{4} \text{ \& } m_2 = \frac{F}{6}$

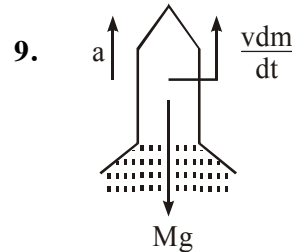
$$a = \frac{F}{m_1 + m_2} = \frac{F}{\frac{F}{4} + \frac{F}{6}} = 2.4 \text{ m/s}^2$$

8. $F \Delta t$ = change in momentum

$$|200(0.25)| = |0 - mv|$$

$$\Rightarrow mv = 50 \text{ kg-ms}^{-1}$$

Hence option (1)



$$\frac{v dm}{dt} - Mg = Ma$$

$$28 \times 10^2(x) = M(a + g)$$

$$= 25(2g)$$

$$= 490$$

$$\Rightarrow x = 0.175 \text{ kg/s}$$

Hence option (3)

10. $a = \frac{5g}{10} = 5 \text{ m/s}^2$ & $v = 0 + 5 \times 4 \Rightarrow v = 20 \text{ m/s}$

Hence option (2)

11. $M_0 - \frac{\Delta m}{\Delta t} \cdot t$ Hence option (1)

12. $F = v \frac{dm}{dt} = 20 \times \frac{50}{60} = 16.66 \text{ N}$

Hence option (4)

13. ΔP = Impulse = mgt

Hence option (3)

14. For min. $\frac{dI}{dt} = 0$

$$40t - 40 = 0$$

$$\Rightarrow t = 1 \text{ sec. Hence option (2)}$$

15. $p = F(0.5) \Rightarrow F = 2p \text{ N}$

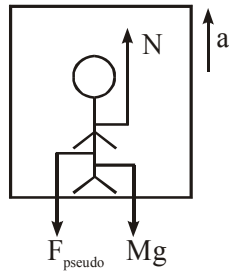
Hence option (1)

NEWTON'S LAWS OF MOTION : RACE # 02

SOLUTION

1.

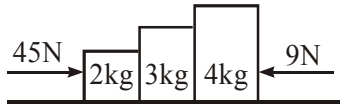
$$\begin{aligned} \text{So } W_{\text{eff}} &= M(g + a) \\ &= 60(14) \\ &= 840 \text{ N} \end{aligned}$$



2.

$$\begin{aligned} M(g + a) &= W_{\text{eff}} \Rightarrow 50(10 + a) = 600 \\ \Rightarrow a &= 2 \text{ m/s}^2 \end{aligned}$$

3.



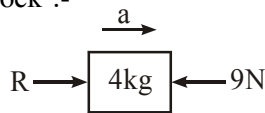
$$a = \frac{45 - 9}{2 + 3 + 4} = 4 \text{ m/s}^2$$

From F.B.D. of 4 kg Block :-

$$R - 9 = ma$$

$$R = 9 + (4 \times 4)$$

$$R = 25 \text{ N}$$



4.

When person dips finger in the water, water will apply upthrust & finger will apply same force on water in downward direction so Net force in downward direction on water increases i.e. reading of balance increases.

Hence option (1)

5.

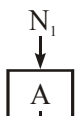
$$a = \frac{600 - 400}{40} = 5 \text{ m/s}^2$$

Hence option (1)

6.



$$N_1 = m_1g$$



$$N_2 = N_1 + m_2g$$

$$N_1 = (m_1 + m_2)g$$

Hence option (2)

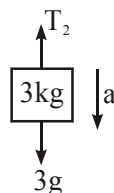


7.

$$a = \frac{3g + 3g - 4g}{3 + 3 + 4} = \frac{g}{5} = 2 \text{ m/s}^2$$

$$\Rightarrow 3g - T_2 = 3 \times a$$

$$\Rightarrow T_2 = 24 \text{ N}$$

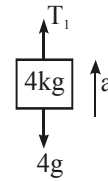


$$\Rightarrow T_1 - 4g = 4a$$

$$T_1 = 48 \text{ N}$$

$$T' = 2T_1$$

$$T' = 96 \text{ N}$$



8.

$$a = \frac{6g - g}{6 + 3 + 1} = \frac{g}{2} = 5 \text{ m/s}^2$$

$$\Rightarrow 6g - T_1 = 6a$$

$$\Rightarrow T_1 = 30 \text{ N}$$

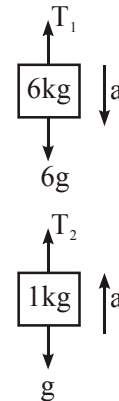
$$T'_1 = T_1 \sqrt{2}$$

$$T'_1 = 30\sqrt{2} \text{ N}$$

$$\Rightarrow T_2 - g = a$$

$$T_2 = 15 \text{ N}$$

$$T'_2 = 15\sqrt{2} \text{ N}$$



9.

$$a = \frac{2mg \sin \theta}{2m + m} = \frac{2}{3} g \sin \theta$$

$$T = ma = \frac{2}{3} mg \sin \theta$$

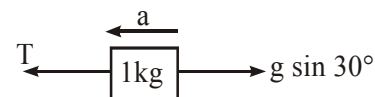
10.

$$a = \frac{8g \sin 30^\circ}{8 + 2} = 4 \text{ m/s}^2$$

$$T = 2a = 8 \text{ N}$$

11.

$$a = \frac{4g \sin 30^\circ - g \sin 30^\circ}{4 + 1} = 3 \text{ m/s}^2$$



$$T - g \sin 30^\circ = 1a \Rightarrow T = 8 \text{ N}$$

12.

$$a_{\text{Train}} = \frac{4 \times 10^5}{80 \times 5 \times 10^3} = 1 \text{ m/s}^2$$

So Tension between 30th & 31st coupling

$$= 50 \times 5 \times 10^3 \times a = 2.5 \times 10^5 \text{ N}$$

Hence option (4)

13.

$$a_{\text{system}} = \frac{F}{2M} \text{ so } T = \left(M + \frac{3}{4}M \right) a$$

$$T = \frac{7}{4} M \left(\frac{F}{2M} \right) \Rightarrow T = \frac{7F}{8}$$

Hence option (1)

14.

$$T - W = ma \Rightarrow 5Mg - Mg = Ma$$

$$\Rightarrow a = 4g$$

Hence option (1)

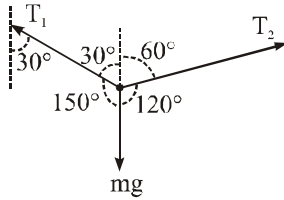
15.

In tug of war contest, the winner has to exert greater force on the ground.

NEWTON'S LAWS OF MOTION : RACE # 03

SOLUTION

1. By Lami's Theorem

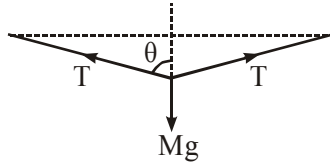


$$\frac{T_1}{\sin 120^\circ} = \frac{mg}{\sin 90^\circ} = \frac{T_2}{\sin 150^\circ}$$

$$T_1 = \frac{mg\sqrt{3}}{2}, T_2 = \frac{mg}{2}$$

2. $2T \cos \theta = Mg$

$$T = \frac{Mg}{2 \cos \theta}$$

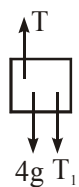


Since θ is very large close to 90°

$T > Mg$

Hence option (2)

3. $\Rightarrow T = 4g + T_1$

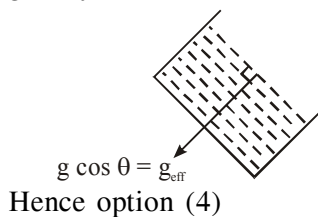


$$\& T = 6g$$

$$\text{So } T_1 = 2g = 20 \text{ N}$$

Hence option (4)

4. The surface will always be $\perp$ to the effective gravity.



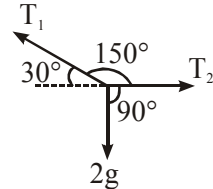
5. $T_2 \sin \theta = T_1 \sin \theta \Rightarrow T_2 = T_1$ & $T_2 = T_3$
Hence option (4)

$$6. \frac{T_1}{\sin 90^\circ} = \frac{2g}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ}$$

$$\Rightarrow T_2 = 2\sqrt{3}g \text{ N}$$

$$= 2\sqrt{3} \text{ kg-wt}$$

Hence option (3)



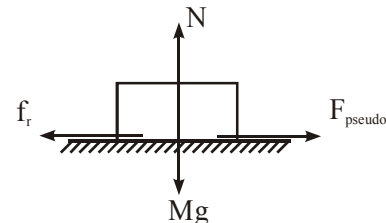
7. $u_1 = 90 \times \frac{5}{18} \text{ m/s} = 25 \text{ m/s}$

$$u_2 = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

$$a = \frac{u_2 - u_1}{\Delta t}$$

$$\text{'a' of truck} = \frac{15 - 25}{10} = -1 \text{ m/s}^2$$

$$\text{so for block } F_{\text{ps}} = f_r = ma$$



$$f_r = F_{\text{ps}} = 500 (-1) = -500 \text{ N}$$

Hence option (1)

FRICITION

SOLUTION

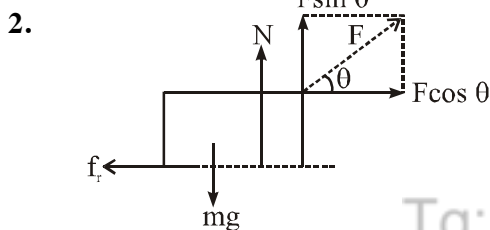
1. $f_s = \mu_s N = 0.2 g = 2N$,
 $f_k = \mu_k N = 0.15 g = 1.5 N$
 (i) When $F = 1 N$
 friction $f_s = 1N \Rightarrow a = 0$
 (ii) When $F = 2N$
 friction $f_s = 2 N$

$$a = \frac{2 - 2}{1} = 0 \text{ m/s}^2$$

 (iii) When $F = 2.5 N$

$$\text{friction } f_k = 1.5 N \Rightarrow a = \frac{F - f_k}{m} = \frac{2.5 - 1.5}{1}$$

$$a = 1 \text{ m/s}^2$$



For limiting condition

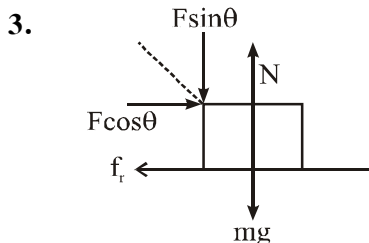
$$f_r = (f_r)_{\text{static}} = F_{\text{applied}}$$

$$\Rightarrow F \cos \theta = \mu N \text{ \& } N = mg - F \sin \theta$$

$$\Rightarrow F \cos \theta = \mu mg - \mu F \sin \theta$$

$$\Rightarrow F (\cos \theta + \mu \sin \theta) = \mu mg$$

$$\text{or } F = \frac{\mu mg}{\cos \theta + \mu \sin \theta}$$



For limiting condition

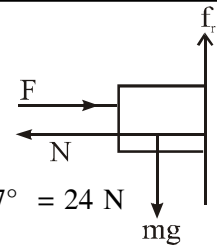
$$(f_r)_{\text{static}} = F_{\text{applied}}, F \cos \theta = \mu N$$

$$N = mg + F \sin \theta$$

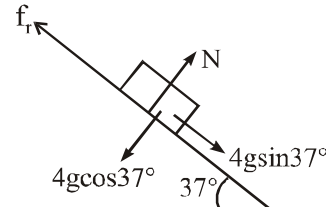
$$\Rightarrow F \cos \theta = \mu mg + \mu F \sin \theta$$

$$\Rightarrow F = \frac{\mu mg}{(\cos \theta - \mu \sin \theta)}$$

4. $N = F \text{ \& } f_r = mg$
 $\Rightarrow \mu F = mg$
 $\Rightarrow F = \frac{mg}{\mu}$



5. Driving force $= 4g \sin 37^\circ = 24 N$



(i) Maximum static friction $= \mu N$
 $= \mu 4g \cos 37^\circ$
 $= 0.4 (32)$
 $= 12.8 N = \text{Friction force acting on block}$

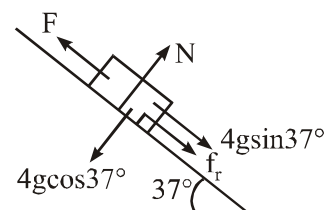
(ii) $a = \frac{24 - 12.8}{4} = 2.8 \text{ m/s}^2$

(iii) To prevent the downward sliding

$$4g \sin 37^\circ = f + F$$

$$\Rightarrow F = 24 - 12.8 \Rightarrow F = 11.2 N \text{ (min value)}$$

(iv) To move upward



$$F = 4g \sin 37^\circ + f = 24 + 12.8$$

$$F = 36.8 N$$

6. $F > f_{\text{limiting}}, F > \mu Mg$
 $4t > 0.4 (10 g)$
 $t > 10 \text{ sec.}$

Hence option (1)

7. Since block is in motion, friction will be $\mu mg = 60 N$
 Hence option (2)

8. $F - f_k = ma$
 $40 - \mu_k (10 \text{ g}) = 10(2) \Rightarrow \mu_k = 0.2$
 Hence option (1)
9. $\mu_k mg \cos \theta = mg \sin \theta \Rightarrow \mu_k = \tan 37^\circ = 3/4$
 Hence option (2)
10. $\frac{g \sin \theta}{g \sin \theta - \mu g \cos \theta} = \frac{2}{1}, \theta = 45^\circ$

$$\frac{1}{1 - \mu} = \frac{2}{1} \Rightarrow \mu = 0.5$$

 Hence option (1)
11. Since acceleration of 'A' is zero, friction force is zero
 Hence option (1)

Tg: @Chalnaayaaar

WORK, POWER & ENERGY: RACE # 01

SOLUTION

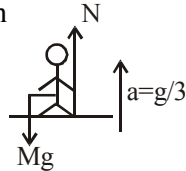
1. Work done by $F_2 = Fx \cos \theta$
 $= 5 (5) (\cos 180^\circ)$
 $= -25 \text{ J}$

Hence option (3)

2. For the motion of person

$$N - Mg = Ma$$

$$\Rightarrow N = \frac{4}{3}Mg$$



work done by normal reaction = $N \times \text{displacement}$

$$= \frac{4}{3} \times 50g \times 12$$

$$= 8000 \text{ J}$$

Hence option (4)

3. Work done against gravity = $mg h/2$

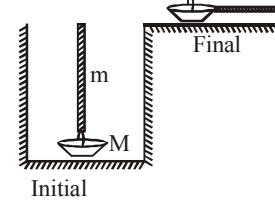
$$= \rho Vg h/2$$

$$= 10^3 \times 1 \times 10 \times \frac{1}{2}$$

$$= \frac{10^4}{2} = 5000 \text{ J}$$

Hence option (1)

4. Work done by the man = change in potential energy
 CMB $\rightarrow$ centre of mass of bucket
 CMR $\rightarrow$ centre of mass of rope



$$= Mg h_{\text{CMB}} + mg h_{\text{CMR}}$$

$$= Mgh + mg \frac{h}{2}$$

$$= \left(M + \frac{m}{2} \right) gh$$

Hence option (3)

5. $U = ax + by$

$$\vec{F} = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} = -a\hat{i} - b\hat{j}$$

$$|\vec{F}| = \sqrt{a^2 + b^2} \text{ so acceleration} = \frac{\sqrt{a^2 + b^2}}{m}$$

Hence option (1)

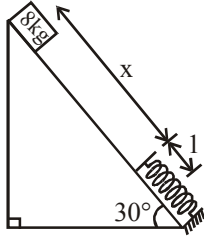
6. $F = \frac{-dU}{dx} = -(\text{slope of } U\text{-}x \text{ curve})$

Hence option (1)

WORK, POWER & ENERGY: RACE # 02

SOLUTION

1. From mechanical energy conservation
change in potential energy = energy stored
in the spring



$$\Rightarrow 8g(x+1)\sin 30^\circ = \frac{1}{2}k(1)^2$$

$$\Rightarrow 100 = 40(x+1) \Rightarrow x = 1.5\text{m}$$

so total distance travelled = 2.5 m

Hence option (1)

2. $x = 3t^2 \Rightarrow v = \frac{dx}{dt} = 6t$ & $a = \frac{dv}{dt} = 6$

$$P = \vec{F} \cdot \vec{v} = m a v = 2 \times 6 \times 6t$$

$$P = 72 \times 4 = 288 \text{ W}$$

Hence option (1)

3. Change in potential energy = work done by
frictional force

$$0 - mgx \sin \theta = -\mu mg(d)$$

$$\Rightarrow \mu = \frac{x \sin \theta}{d} \quad \text{Hence option (3)}$$

4. Work done by the force = change in K.E.

$$\Rightarrow \frac{1}{2}mv^2 = \int F dx = \text{Area under } F - x \text{ curve}$$

$$\Rightarrow \frac{1}{2}(0.1)v^2 = \frac{1}{2}(12+4) \times 10 \Rightarrow v^2 = 1600$$

$$\Rightarrow v = 40 \text{ m/s} \quad \text{Hence option (4)}$$

5. Work done = change in kinetic energy
for minimum time used power should be
maximum i.e. P.

$$\text{So, } Pt = \frac{1}{2}mv^2 \Rightarrow t = \frac{mv^2}{2P}$$

Hence option (1)

6. $P = 2x$

$$\Rightarrow F \cdot v = 2x \Rightarrow mav = 2x$$

$$\Rightarrow m \cdot \frac{v dv}{dx} \cdot v = 2x$$

$$\Rightarrow v^2 dv = \frac{2}{m} x dx$$

on integrating both sides

$$\Rightarrow \frac{v^3}{3} = \frac{2}{m} \frac{x^2}{2} \Rightarrow v = \left[\frac{3x^2}{m} \right]^{1/3}$$

Hence option (1)

CIRCULAR MOTION

SOLUTION

1. According to question $\frac{mv^2}{r} = \frac{K}{r^2} \Rightarrow K.E. = \frac{1}{2}mv^2 = \frac{K}{2r}$

$$P.E. = -\int_{\infty}^r \vec{F} \cdot d\vec{r} = -\int_{\infty}^r \left(-\frac{K}{r^2} \hat{r}\right) \cdot d\vec{r} = \int_{\infty}^r \frac{K}{r^2} dr = -\frac{K}{r}$$

$$T.E. = K.E. + P.E. = \frac{K}{2r} + \left(-\frac{K}{r}\right) = \frac{-K}{2r}$$

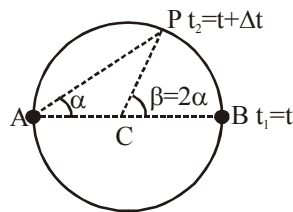
2. For motion to be circular a_r must be non zero,
 a_t may or may not be zero.
 $a_r \approx 0$

3. $\omega_A = \frac{\alpha}{\Delta t}$

$$\omega_C = \frac{\beta}{\Delta t} = \frac{2\alpha}{\Delta t}$$

$$\omega_C = 2\omega_A$$

$$\boxed{\frac{\omega_A}{\omega_C} = \frac{1}{2}}$$



4. $a_t = 4m/s^2$

$$a_c = \frac{v^2}{r} = 3m/s^2$$

$$a_{net} = \sqrt{a_t^2 + a_c^2} = 5m/s^2$$

5. Total acceleration $a = \sqrt{a_c^2 + a_t^2}$

$$5 = \sqrt{\left(\frac{v^2}{r}\right)^2 + 3^2} = \sqrt{\left(\frac{12^2}{r}\right)^2 + 3^2}$$

$$\Rightarrow r = 36m$$

6. $a_t = \frac{dv}{dt} = a$

$$\omega^2 = \omega_0^2 + 2\alpha\theta = 0^2 + 2\left(\frac{a}{R}\right)\left(\frac{2\pi}{8}\right) = \frac{\pi a}{2R}$$

$$\therefore a_c = \omega^2 R = \frac{\pi a}{2}$$

$$a_{net} = \sqrt{a_t^2 + a_c^2}$$

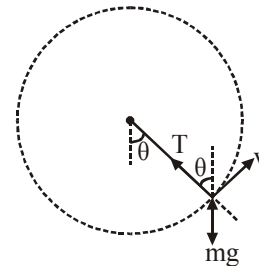
$$a_{net} = \frac{a}{2} \sqrt{4 + \pi^2}$$

8. $a_c = 9g$
 $(2\pi n)^2 \cdot r = 9g$

$$n = \frac{1}{2\pi} \sqrt{\frac{9g}{r}} \quad \boxed{n = 0.675 \frac{\text{rev}}{\text{sec}}}$$

9. $v_{max.} = \sqrt{\mu Rg} = \sqrt{(0.6)(150)(10)} = 30m/s$

10. $T - mg \cos \theta = \frac{mv^2}{\ell}$



$$\Rightarrow 51.6 - 2g \cos \theta = \frac{2(4)^2}{1}$$

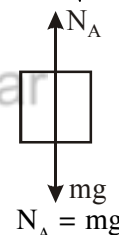
$$\Rightarrow \cos \theta = 1$$

$$\Rightarrow \theta = 0^\circ$$

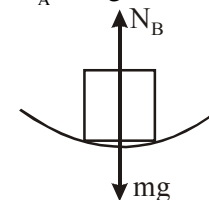
11. To leave the hemisphere without sliding down
 $N = 0$

So $mg = \frac{mv^2}{R}$
 $\Rightarrow v = \sqrt{Rg}$

12. At A



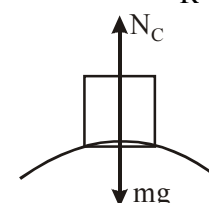
At B



$$N_B - mg = \frac{mV^2}{R}$$

$$N_B = mg + \frac{mV^2}{R}$$

At C



$$mg - N_C = \frac{mV^2}{R}$$

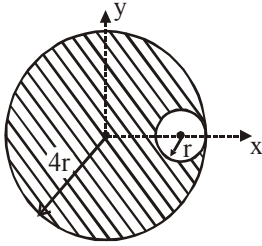
$$N_C = mg - \frac{mV^2}{R}$$

$$N_B > N_A > N_C$$

COLLISION AND CENTRE OF MASS

SOLUTION

1.



Let mass of larger disc = M

then mass of smaller disc of radius $r = \frac{M}{16}$

$$x_{cm} = \frac{M(0) + (-M/16)(3r)}{M + (-M/16)} = -\frac{r}{5}$$

i.e. centre of mass of the residual disc will be at a distance $r/5$ from centre of larger disc.

2.
$$\vec{a}_{cm} = \frac{m_1(\vec{g}) + m_2(\vec{g})}{m_1 + m_2} = \vec{g} = g \text{ downward}$$

3. Let the person slips x distance then $m_1 x_1 = m_2 x_2$

$$\Rightarrow \frac{M}{2}(x) = M(10-x)$$

$$\Rightarrow 3x = 20$$

$$\Rightarrow x = \frac{20}{3} \text{ m}$$

4. Velocity of centre of mass will be zero as the net force on the system is zero. so centre of mass will remain in its original position.

5. Velocity of centre of mass

$$V_{cm} = \frac{5 \times 7 + 2 \times 0}{5 + 2} = 5 \text{ m/s}$$

6. Work done = change in kinetic energy

$$= \left(\frac{1}{2} m_1 u^2 + \frac{1}{2} m_2 v^2 \right) - 0$$

[On applying conservation of linear momentum]

$$[m_1 u - m_2 v = 0]$$

$$= \frac{1}{2} m_1 u^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2} u \right)^2$$

$$= \frac{1}{2} m_1 u^2 \left[1 + \frac{m_1}{m_2} \right]$$

7. From conservation of linear momentum

$$2 \times 3 - 1 \times 4 = (2 + 1)v$$

$$v = \frac{2}{3} \text{ m/s}$$

8. Height of rebound = $e^2 h = (0.6)^2 (1) = 0.36 \text{ m}$



Before collision After collision

After collision velocity of heavy body doesn't change

by definition of 'e', $e = 1 = \frac{v_1 - v}{v - 0} \Rightarrow v_1 = 2v$

ROTATIONAL MOTION : RACE # 01

SOLUTION

1. $\omega_0 = \frac{450 \times 2\pi}{60} \text{ rad/sec.} = 15\pi \text{ rad/s}$
 $t = 10 \text{ sec.}$
 $\omega_f = 0 \text{ rad/sec.}$
 $\theta = \frac{1}{2}t(\omega_f + \omega_0) = \frac{1}{2} \times 10 \times (15\pi + 0) = 75\pi$
 $\text{No. of revolutions} = \frac{\theta}{2\pi} = \frac{75\pi}{2\pi} = 37.5$
Hence option (1)
2. $\theta = \frac{1}{2}t(\omega_i + \omega_f)$
 $= \frac{1}{2} \times 4 (0 + 100 \times 2\pi) = 400\pi$
Hence option (4)
3. $\theta = \frac{1}{2}t(\omega_i + \omega_f) \Rightarrow 100\pi = \frac{1}{2}(5)[0 + \omega]$
 $\Rightarrow \omega = 40\pi \text{ Rad/sec.}$
Hence option (1)
4. For acceleration
 $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 2 \times (20)^2 = 400 \text{ Rad.}$
 $\omega = 0 + \alpha t = 2 \times 20 = 40 \text{ Rad/sec}$
For uniform motion
 $\theta = \omega \times t = 40 \times 10 = 400 \text{ Rad.}$
For retardation
 $\theta = \frac{1}{2}t(\omega_i + \omega_f) = \frac{1}{2} \times 20 (40 + 0) = 400 \text{ Rad.}$
Total angle of rotation = 1200 Rad.
Hence option (2)
5. $\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{20^2 - 10^2}{2 \times 3} = 50 \text{ Radian}$
Hence option (1)
6. $I = \frac{MR^2}{2} = \frac{\rho(\pi R^2)xR^2}{2}$ so $I \propto R^4$
(where x is thickness)
Hence option (2)

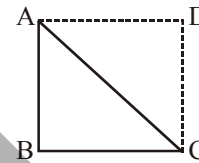
$$7. I = \frac{Mr^2}{2} \quad I_{\text{ring}} = n^2 Mr^2 = 2n^2 I$$

$$8. I = MR^2 = \rho_\ell (2\pi R)R^2$$

$$\Rightarrow I \propto R^3 \text{ so } \frac{I_A}{I_B} = \left(\frac{R_A}{R_B}\right)^3 = \frac{1}{8}$$

Hence option (2)

9. The moment of Inertia of triangular sheet about AC will be half the moment of inertia of square plate about its diagonal AC

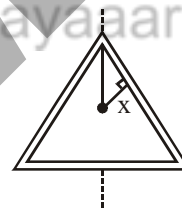


$$I_{AC} = \frac{1}{2} \times \frac{(2M)L^2}{12} = \frac{ML^2}{12}$$

Hence option (1)

10. $K = \sqrt{\frac{I}{m}}, K \propto \sqrt{I}$
and I changes with axis of rotation.

11.



$$I = 3 \times \left\{ \frac{mL^2}{12} + mx^2 \right\}$$

$$\text{here } x = \frac{1}{3}L \sin 60^\circ = \frac{L}{2\sqrt{3}}$$

$$\text{so } I = 3 \times \left\{ \frac{mL^2}{12} + m \frac{L^2}{12} \right\} = 3 \frac{mL^2}{6} = \frac{mL^2}{2}$$

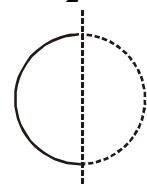
Hence option (2)

12. Moment of inertia of semicircular portion is half of the total ring

$$\text{so } I = \frac{1}{2} M_{\text{ring}} \frac{R^2}{2} = \frac{1}{2} (2m) \frac{R^2}{2} = \frac{mR^2}{2}$$

$$\& \pi R = \ell \text{ so } R = \frac{\ell}{\pi}$$

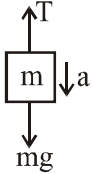
$$I = \frac{m\ell^2}{2\pi^2} \text{ Hence option (4)}$$



ROTATIONAL MOTION : RACE # 02

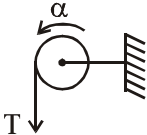
SOLUTION

1. FBD of Block



$$mg - T = ma \quad \dots(1)$$

FBD of Disc



$$T.R = I.\alpha$$

$$T.R = I.\frac{a}{R} \quad (\because a = R\alpha)$$

$$T = I\frac{a}{R^2} \quad \dots(2)$$

from eqⁿ (1) & (2)

$$mg - I\frac{a}{R^2} = ma$$

$$a = \frac{mg}{m + \frac{I}{R^2}}$$

Hence, acceleration of the block = $\frac{mg}{m + \frac{I}{R^2}}$

$$= \frac{mg}{m + \frac{m}{2}} = \frac{2}{3}g$$

so velocity $v = \sqrt{2ah}$

$$= \sqrt{\frac{4gh}{3}} = 2\sqrt{\frac{gh}{3}}$$

Hence option (3)

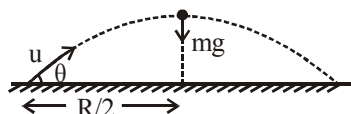
2. Torque = Force $\times$ perpendicular distance from the reference point

$$\tau = mg \frac{(R)}{2}$$

$$= mg \frac{u^2}{2g} \sin 2\theta$$

$$= mu^2 \sin\theta \cos\theta$$

Hence option (2)



3. $\vec{\tau} = \vec{r} \times \vec{F}$

$$= (\hat{i} - 2\hat{j} + \hat{k}) \times (-2\hat{i} + 2\hat{j} + 3\hat{k})$$

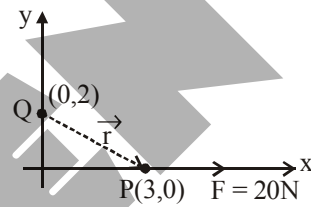
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ -2 & 2 & 3 \end{vmatrix}$$

$$= \hat{i}(-6 - 2) - \hat{j}\{3 - (-2)\} + \hat{k}(2 - 4)$$

$$= -8\hat{i} - 5\hat{j} - 2\hat{k}$$

Hence option (2)

4.



torque = force $\times$ perpendicular distance

$$= 20 \times 2$$

$$= 40 \text{ N-m} \quad \text{Hence option (3)}$$

5. Power = $\vec{\tau} \cdot \vec{\omega}$

$$= (\vec{r} \times \vec{F}) \cdot \vec{\omega} \quad \text{Hence option (1)}$$

6. $\tau = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{const.} \Rightarrow \vec{r} \times \vec{p} = \text{const.}$

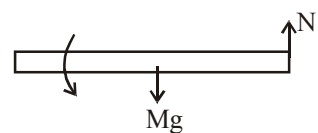
7. For rotation $Mg(\ell/2) = \frac{M\ell^2}{3}\alpha$

$$\Rightarrow \alpha = \frac{3g}{2\ell}$$

$$\text{so } a_{\text{cm}} = \frac{\ell}{2} \alpha = \frac{3g}{4}$$

$$\text{so } Mg - N = M\left(\frac{3g}{4}\right) \Rightarrow N = \frac{Mg}{4} = \frac{W}{4}$$

Hence option (1)



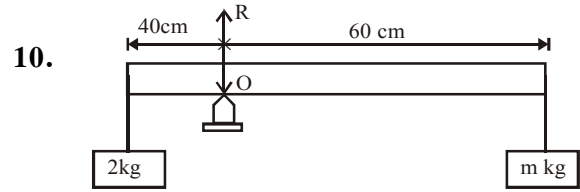
8. $\tau = I \alpha \Rightarrow \alpha = \frac{\tau}{I}$

$$\alpha = \frac{(F \sin 30^\circ)L}{\frac{ML^2}{3}} = \frac{3F}{2ML}$$

Hence option (1)

9. $\alpha = \frac{\tau}{I} = \frac{2FR}{MR^2} = \frac{4F}{MR}$

Hence option (3)



For balancing

$R = (2 + m)g$ & torque about pivot is zero

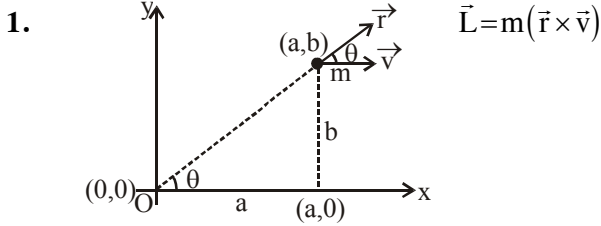
$$2g(0.4) - mg(0.6) = 0$$

$$\Rightarrow m = \frac{8}{6} = \frac{4}{3} \text{ kg} \quad \& \quad R = \frac{10}{3}g = \frac{100}{3} \text{ N}$$

Tg: @Chalnaayaaar

ROTATIONAL MOTION : RACE # 03

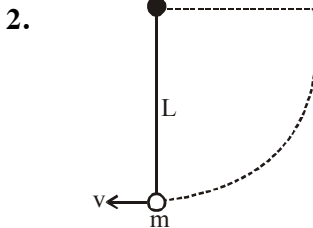
SOLUTION



$$|\vec{L}| = mv r_{\perp} = mvb$$

Direction : $-\hat{k}$ (Right Hand thumb Rule)

$$\Rightarrow \vec{L} = -mvb\hat{k}$$



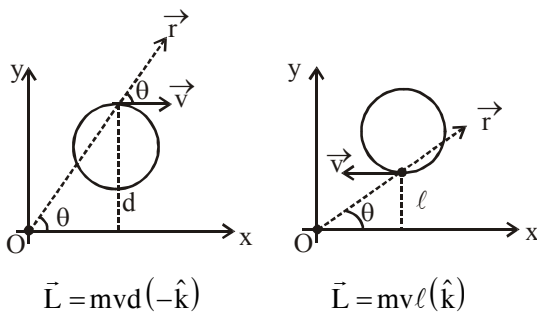
from energy conservation for bob :-

$$K_i + U_i = K_f + U_f$$

$$0 + mgL = \frac{1}{2}mv^2 + 0 \Rightarrow v = \sqrt{2gL}$$

$$L = mv r_{\perp} = m\sqrt{2gL} \cdot L = mL \sqrt{2gL}$$

3. As particle moves with constant speed but the magnitude & direction of angular momentum changes continuously. As shown in figure because perpendicular distance of the particle and $\vec{v}$ changes continuously.



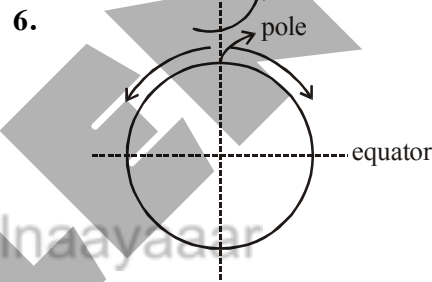
$L = 0$ because angle between $\vec{r}$ & $\vec{v}$ is always zero (as point lies on the same line)

5. From conservation of angular momentum

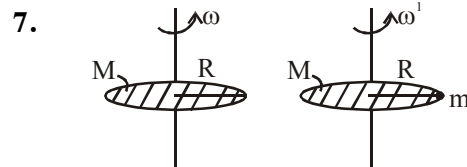
$$I_1 \omega_1 = I_2 \omega_2$$

$$\frac{2}{5}MR^2 \frac{2\pi}{T_1} = \frac{2}{5}M\left(\frac{R}{2}\right)^2 \frac{2\pi}{T_2}$$

$$T_2 = \frac{T_1}{4} = \frac{24}{4} = 6 \text{ hr}$$



After melting the ice on polar caps it reaches towards equator (i.e. away from the axis) Then I increases and ω decreases because angular momentum is conserved.



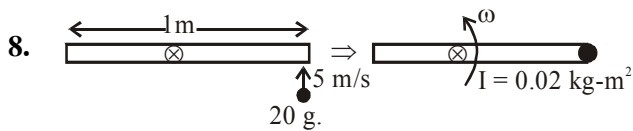
from conservation of angular momentum

$$I\omega = I'\omega'$$

$$\Rightarrow \frac{MR^2}{2}\omega = \left(\frac{MR^2}{2} + mR^2\right)\omega'$$

$$\Rightarrow \frac{1 \times (0.1)^2}{2} \times 20 = \left[\frac{1 \times (0.1)^2}{2} + 0.5 \times (0.1)^2\right]\omega'$$

$$\Rightarrow \omega' = 10 \text{ rad/s}$$



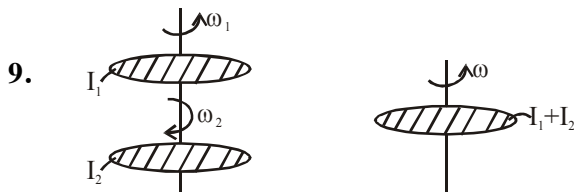
from conservation of angular momentum

$$L_i = L_f$$

$$mvr = I_{\text{total}} \omega.$$

$$0.020 \times 5 \times \frac{1}{2} = (0.02) \omega.$$

$$\omega = 2.5 \text{ rad / sec.}$$

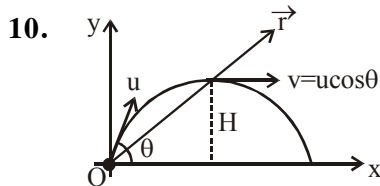


from conservation of angular momentum

$$L_i = L_f$$

$$\Rightarrow I_1 \omega_1 - I_2 \omega_2 = (I_1 + I_2) \omega$$

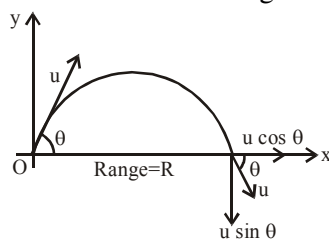
$$\Rightarrow \omega = \frac{I_1 \omega_1 - I_2 \omega_2}{I_1 + I_2}$$



$$L = m v r_{\perp}$$

$$= mu \cos \theta H$$

$$= (mu \cos \theta) \frac{u^2 \sin^2 \theta}{2g} = \frac{mu^3 \cos \theta \sin^2 \theta}{2g}$$



$$L = mvr_{\perp}$$

$$= mu \sin \theta R = \frac{m u \sin \theta u^2 \sin 2\theta}{g}$$

$$\Rightarrow L = \frac{2mu^3 \sin^2 \theta \cos \theta}{g}$$

11. Angular Impulse = change in Angular momentum

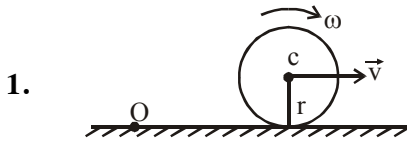
$$20 = I(\omega_f - \omega_i) \Rightarrow \Delta\omega = \frac{20}{I} = \frac{20}{MR^2}$$

$$\Delta\omega = \frac{20}{2 \times (0.2)^2} = \frac{10}{0.04} = 250 \text{ rad/sec.}$$

Hence option (3)

ROTATIONAL MOTION : RACE # 04

SOLUTION

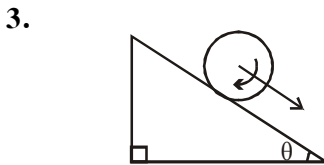


1. Sphere is in pure rolling condition i.e. $v = r\omega$.

$$\begin{aligned} L_0 &= L_{CM} + L_{\text{of body about CM}} \\ &= mvr + I\omega \\ &= mvr + \frac{2}{5}mr^2\left(\frac{v}{r}\right) = \frac{7}{5}mvr \end{aligned}$$

2. For pure rolling velocity of contact point on the object w.r.t. the contact point on the surface should be zero. And friction always tries to stop the relative motion hence can act in either direction

Hence option (4)



$$a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}}$$

Here $\theta = 30^\circ$; I for ring $= MR^2$

$$\text{so } a = \frac{g \sin 30^\circ}{1 + 1} = \frac{g}{4} \quad \text{Hence option (4)}$$

4. For an object rolling down the plane

$$a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}} \quad \text{where } K \rightarrow \text{Radius of gyration}$$

$$\text{time taken } t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2S \left(1 + \frac{K^2}{R^2}\right)}{g \sin \theta}}$$

clearly, time is independent of mass, Radius & density Hence option (4)

$$5. \quad \text{Time for slipping } t_s = \sqrt{\frac{2S}{g \sin \theta}}$$

$$\text{time for rolling } t_R = \sqrt{\frac{2S \left(1 + \frac{K^2}{R^2}\right)}{g \sin \theta}}$$

$$\text{so } \frac{t_s}{t_R} = \sqrt{\frac{1}{1 + \frac{K^2}{R^2}}}$$

$$\text{For ring } \frac{K^2}{R^2} = 1$$

$$\text{so } \frac{t_s}{t_R} = \frac{1}{\sqrt{2}} \quad \text{Hence option (3)}$$

6. For rolling upon the incline plane the retardation

$$a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}} = \frac{10 (\sin 30^\circ)}{1 + \frac{2}{5}} = 25/7 \text{ m/s}^2$$

$$\text{so distance covered} = \frac{v^2}{2a} = \frac{(2)^2}{2 \times 25/7} = 0.56 \text{ m} = 56 \text{ cm}$$

Hence option (1)

7. Ans. (1)

$$\frac{\text{Rotational K.E.}}{\text{Total K.E.}} = \frac{2}{5} \Rightarrow \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} = \frac{2}{5}$$

$$\Rightarrow \frac{\frac{1}{2}mK^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{2}mK^2\omega^2} = \frac{2}{5}$$

$$\Rightarrow \frac{K^2}{K^2 + R^2} = \frac{2}{5} \Rightarrow K^2 = \frac{2}{3}R^2 \quad \text{or } K = \sqrt{\frac{2}{3}}R$$

so it will be hollow sphere

Hence option (1)

8. Velocity after travelling distance 's' $v = \sqrt{2as}$

$$\text{so } \frac{v_s}{v_R} = \sqrt{\frac{a_s}{a_R}} \Rightarrow v_R = \sqrt{\frac{a_R}{a_s}} v_s$$

$$a_R = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}} = \frac{g \sin \theta}{1 + \frac{I}{MR^2}} = \frac{g \sin \theta}{1 + \frac{1}{2}} = \frac{2}{3} g \sin \theta$$

$$a_s = g \sin \theta \Rightarrow \frac{a_R}{a_s} = \frac{2}{3}$$

$$\text{so, } v_R = v \sqrt{\frac{2}{3}} \quad \text{Hence option (1)}$$

9. $\mu_{\min} = \frac{\tan \theta}{1 + \frac{R^2}{K^2}}$

For solid cylinder

$$\frac{K^2}{R^2} = \frac{1}{2}$$

$$\mu_{\min} = \frac{1}{3} \tan \theta \quad \text{Hence option (1)}$$

10. $\frac{\text{Rotational kinetic energy}}{\text{Translational kinetic energy}} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} M v^2}$

$$= \frac{I}{M R^2} = \frac{2}{5} \quad (\text{for solid sphere})$$

Hence option (1)

11. $\frac{v_1}{v_2} = \sqrt{\frac{a_1}{a_2}}$

as we know acceleration is independent from mass & Radius so $a_1 = a_2$

$$\Rightarrow v_1 : v_2 = 1 : 1 \quad \text{Hence option (4)}$$

12. $\frac{\text{Rotational KE}}{\text{Rotational KE} + \text{Trans. KE}}$

$$= \frac{1}{\frac{\text{Trans.}}{\text{Rotational}} + 1} = \frac{1}{\left(\frac{R^2}{K^2} + 1\right)}$$

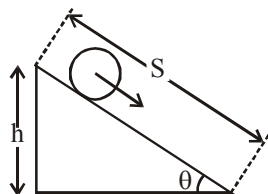
This term will be maximum when K is maximum and here K is maximum for ring.

13. Angular velocity $\omega = \frac{v}{R}$

$$= \frac{\sqrt{2as}}{R}$$

$$= \frac{1}{R} \sqrt{\frac{2g \sin \theta (s)}{1 + \frac{K^2}{R^2}}}$$

$$= \frac{1}{R} \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}}$$



$$\therefore \frac{K^2}{R^2} = \frac{1}{2} \quad (\text{for solid cylinder})$$

$$\therefore \omega = \frac{1}{R} \sqrt{\frac{4gh}{3}} = \frac{2}{R} \sqrt{\frac{gh}{3}} \quad \text{Hence option (3)}$$

14. $K = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 = \frac{1}{2} m v^2 \left(1 + \frac{K^2}{R^2}\right)$

$$= \frac{1}{2} m v^2 \left[1 + \frac{2}{5}\right]$$

$$= \frac{7}{10} m v^2 = 56 \text{ J}$$

Hence option (2)

15. $v = \sqrt{\frac{2gH}{1 + \frac{K^2}{R^2}}}$

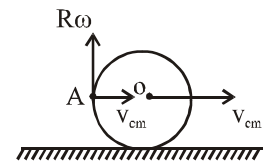
$$= \sqrt{\frac{4gH}{3}} \quad (\text{As } I = \frac{MR^2}{2} \text{ for solid cylinder})$$

Hence option (4)

16.

Velocity at point 'A'

$$= \sqrt{v_{\text{cm}}^2 + R^2 \omega^2}$$



as $v_{\text{cm}} = R\omega$ (For pure rolling)

$$v_A = v_{\text{cm}} \sqrt{2} \quad \text{Hence option (2)}$$

17. As the contact point is relatively stationary so work done by the friction is always zero.

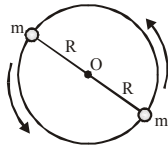
Hence option (1)

18. $a = \frac{g \sin \theta}{1 + \frac{I}{M R^2}} = \frac{g \sin 30^\circ}{1 + \frac{2}{5}} = \frac{5}{14} g$

Hence option (4)

1. Centripetal force is provided by the gravitational force of attraction between two particles i.e.

$$\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2} \Rightarrow v = \frac{1}{2} \sqrt{\frac{Gm}{R}}$$



2. From conservation of mechanical energy

$$\frac{1}{2}mv^2 = \frac{GMm}{R_e} - \frac{GMm}{R}$$

Where R = maximum distance from centre of

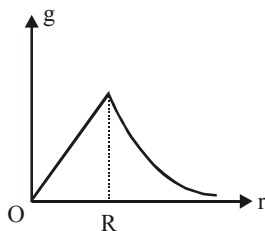
the earth Also $v = \frac{1}{4}v_e = \frac{1}{4}\sqrt{\frac{2GM}{R_e}}$

$$\Rightarrow \frac{1}{2}m \times \frac{1}{16} \times \frac{2GM}{R_e} = \frac{GMm}{R_e} - \frac{GMm}{R}$$

$$\Rightarrow R = \frac{16}{15}R_e$$

$$\Rightarrow h = R - R_e = \frac{R_e}{15}$$

3. Variation of g with distance



variation of g with ω : $g' = g - \omega^2 R \cos^2 \lambda$

If $\omega=0$ then g will not change at poles where $\cos \lambda = 0$.

4. $I_g = \frac{GM}{R^2}$, $V = -\frac{GM}{R}$,

$$V = -I_g R = -6 \times 8 \times 10^6$$

$$= -4.8 \times 10^7 \text{ newton-meter/kg}$$

6. If gravitational force becomes zero, then satellite will not follow circular path due to absence of required centripetal force, hence it will move tangential to the orbit.

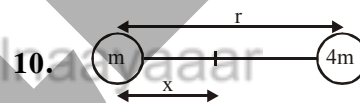
7. Required KE = $\frac{GMm}{R} = mgR$

8. $g_h = g\left(1 - \frac{2h}{R}\right)$; $g_d = g\left(1 - \frac{d}{R}\right)$

$$\Delta g_h = \Delta g_d \Rightarrow g\left(\frac{2h}{R}\right) = g\left(\frac{d}{R}\right) \Rightarrow 2h = d$$

9. $g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = \frac{g}{9} \Rightarrow \left(1 + \frac{h}{R}\right)^2 = 9$

$$\Rightarrow 1 + \frac{h}{R} = 3 \Rightarrow h = 2R$$



10.

$$\frac{Gm}{x^2} = \frac{G4m}{(r-x)^2} \Rightarrow x = \frac{r}{3}, r-x = \frac{2r}{3}$$

Potential at point the gravitational field is zero between the masses.

$$V = \frac{-Gm}{x} - \frac{G(4m)}{(r-x)}$$

$$V = -\frac{3Gm}{r} - \frac{3G \times 4m}{2r}$$

$$= -\frac{3Gm}{r} [1 + 2] = -\frac{9Gm}{r}$$

11. (I) Gravitational force between two particles is independent of presence of other particles. Hence, the force between M and m is F .

(II) Gravitational force on $M = F_1 + F_2$

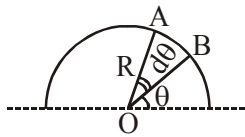
$$= \frac{GMm}{a^2} + \frac{GM(2m)}{a^2} = \frac{3GMm}{a^2} = 3F.$$

$$12. \quad g' = \frac{64}{100}g, \quad g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$\frac{64}{100}g = \frac{g}{\left(1 + \frac{h}{R}\right)^2}, \quad \left(1 + \frac{h}{R}\right)^2 = \frac{100}{64}$$

$$1 + \frac{h}{R} = \frac{5}{4}, \quad \frac{h}{R} = \frac{1}{4}, \quad h = R/4$$

13. Mass per unit length of the rod = $\frac{M}{L}$
Mass of small length element AB,



$$dm = \left(\frac{M}{L}\right) R d\theta$$

Potential at O due to this element,

$$dV = -G \cdot \frac{dm}{R} = -\frac{GM}{L} \frac{R d\theta}{R} = -\frac{GM}{L} d\theta$$

$$\therefore V = \int dV = -\frac{GM}{L} \int_0^\pi d\theta = -\frac{\pi GM}{L}$$

14. Here, angular momentum is conserved.
According to it,
 $V_{\max} R_{\min} = V_{\min} R_{\max}$
At the point P_4 , the value of R is minimum and hence velocity is maximum or KE is maximum

$$15. \quad g_{\text{eff}} = g - \omega^2 R \cos^2 \lambda$$

$$16. \quad \text{Change in PE, } \Delta U = U_2 - U_1$$

$$\therefore \Delta U = -\frac{GMm}{(R+nR)} + \frac{GMm}{R}$$

$$\text{or } \Delta U = -\frac{GMm}{R(1+n)} + \frac{GMm}{R}$$

$$= \frac{GMm}{R} \left[-\frac{1}{1+n} + 1 \right]$$

$$\text{or } \Delta U = \frac{(R^2 g)m}{R} \times \frac{n}{1+n} \left[\because g = \frac{GM}{R^2} \right]$$

$$\text{or } \Delta U = mgR \left(\frac{n}{1+n} \right)$$

17. Angular momentum is conserved
[Net torque = 0]

$$18. \quad v_0 = \sqrt{\frac{GM}{r}}, \quad v_0 \propto \frac{1}{\sqrt{r}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} \quad r_1 = 4R, \quad r_2 = R, \quad v_1 = 3v$$

$$\frac{3v}{v_2} = \sqrt{\frac{R}{4R}} \quad \boxed{v_2 = 6v}$$

Properties of Matter & Fluid Mechanics (Elasticity) : RACE # 01

SOLUTION

1. Stored energy = $W = \frac{1}{2} F \Delta \ell = \frac{1}{2} \frac{F^2 L}{AY}$

$$\frac{W_A}{W_B} = \frac{L_A}{L_B} \times \left(\frac{r_B}{r_A} \right)^2 = 3 \times \left(\frac{1}{2} \right)^2 = \frac{3}{4}$$

2. Breaking stress is independent from radius of wire depend on material of body.

3. $K = \frac{\Delta P}{\left(-\frac{\Delta V}{V} \right)} = \frac{h \rho g}{\left(-\frac{\Delta V}{V} \right)}$

$$9.1 \times 10^8 = \frac{h \times 10^3 \times 10}{\left(\frac{0.1}{100} \right)} \Rightarrow h = 91 \text{ m}$$

4. $B = \frac{\Delta P}{\left(-\frac{\Delta V}{V} \right)} \Rightarrow \left(-\frac{\Delta V}{V} \right) = \frac{\Delta P}{B} = \frac{Mg}{AB}$

For sphere $V = \frac{4}{3} \pi R^3 \Rightarrow \frac{\Delta V}{V} = 3 \left(\frac{\Delta R}{R} \right)$

$$\Rightarrow \left(-\frac{\Delta V}{V} \right) = 3 \left(-\frac{\Delta R}{R} \right) = \frac{Mg}{AB}$$

$$\Rightarrow \left(-\frac{\Delta R}{R} \right) = \frac{Mg}{3AB}$$

5. $K = \frac{\Delta P}{\left(-\frac{\Delta V}{V} \right)} = \frac{(1.165 \times 10^5 - 1.01 \times 10^5)}{\left(\frac{10}{100} \right)}$

$$= \frac{0.155 \times 10^5}{\frac{1}{10}} = 1.55 \times 10^5 \text{ Pa}$$

6. Work = Kinetic energy

$$\frac{1}{2} F \Delta \ell = \frac{1}{2} m v^2$$

$$\frac{1}{2} \frac{AY}{L} (\Delta \ell)^2 = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{AY(\Delta \ell)^2}{Lm}} = \sqrt{\frac{10^{-6} \times 5 \times 10^8 \times (2 \times 10^{-2})^2}{10 \times 10^{-2} \times 5 \times 10^{-3}}}$$

$$= 20 \text{ ms}^{-1}$$

7. If the initial length of the wire is ℓ_0 then

$$M_1 g = K(\ell_1 - \ell_0) \text{ \& } (M_1 + M_2)g = K(\ell_2 - \ell_0)$$

$$\Rightarrow \frac{M_1 + M_2}{M_1} = \frac{\ell_2 - \ell_0}{\ell_1 - \ell_0} \Rightarrow \ell_0 = \frac{M_1}{M_2} (\ell_1 - \ell_2) + \ell_1$$

8. Young's modulus of elasticity is independent from length and radius of wire.

9. $W = \frac{1}{2} F \Delta \ell = \frac{1}{2} (5 \times 10) \times (30 \times 10^{-2}) = 7.5 \text{ J}$

10. $\Delta \ell = \frac{FL}{AY}$

Here $\Delta \ell$, F , L are same

$$A \propto \frac{1}{Y}$$

$$\therefore \frac{R_B^2}{R_S^2} = \frac{Y_S}{Y_B} = \frac{2 \times 10^{10}}{10^{10}} = 2$$

$$R_B = \sqrt{2} R_S$$

or $R_S = \frac{R_B}{\sqrt{2}}$

11. Maximum stress on the wire will be at the highest point (the point of suspension).

$$\text{Stress} = \frac{\text{Weight}}{\text{Area}}$$

or $\sigma = \frac{A \rho g}{A}$

or $l = \frac{\sigma}{\rho g}$

12. Energy per unit volume = $\frac{1}{2} \times \text{stress} \times \text{strain}$

$$= \frac{1}{2} \times Y (\text{strain})^2$$

$$= \frac{1}{2} \times 2 \times 10^{10} \times (10^{-2})^2 = 10^6 \text{ J/m}^3$$

$$13. \quad \eta = \frac{FL}{Ax} = \frac{(8 \times 10^3) \times (40 \times 10^{-2})}{(40 \times 10^{-2})^2 \times 0.1 \times 10^{-3}} = 2 \times 10^8 \text{ Nm}^{-2}$$

14. For incompressible liquid strain = 0
so its bulk modulus is infinity

$$15. \quad Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{max. strain} = \frac{\text{max. stress}}{Y} = \frac{mg / A}{Y}$$

$$\therefore m = \frac{Y \times \text{max. strain} \times A}{g}$$

$$= \frac{2 \times 10^{11} \times 10^{-3} \times 3 \times 10^{-6}}{10} = 60 \text{ kg}$$

$$16. \quad Y = \frac{\text{Stress}}{\text{Strain}}$$

if strain = 1

then Y = stress

Tg: @Chalnaayaaar

Properties of Matter & Fluid Mechanics (Fluid Dynamics + Viscosity) : RACE # 02

SOLUTION

1. Rate of flow $Q = Av$

$$v = \frac{Q}{A} = \frac{200}{0.5 \times 10^4} = 4 \times 10^{-2} \text{ cm/s} = 0.4 \text{ mm/s}$$

2. $A_1 v_1 = A_2 v_2$

$$(8 \times 10^{-4}) \left(\frac{0.15}{60} \right) = (40 \times 10^{-8})(v)$$

$$v = 5 \text{ ms}^{-1}$$

3. For maximum range $h = \frac{H}{2}$

4. Maximum height of water in tank obtained when incoming volume of water is equal to outgoing volume of water

$$Q = A_1 v_1 = A \times \sqrt{2gH}$$

$$70 = 1 \times \sqrt{2 \times 980 \times H}$$

$$H = 2.5 \text{ cm}$$

5. $\Delta P = \frac{1}{2} \rho (v_2^2 - v_1^2)$

$$= \frac{1}{2} \times 1.3 ((120)^2 - (90)^2) = 4095 \text{ Pa}$$

6. $v_T = \frac{2}{9} \frac{r^2 \rho_L g}{\eta}$

$$\eta = \frac{2}{9} \frac{r^2 \rho_L g}{v_T} = \frac{2}{9} \frac{(1)^2 \times 1.5 \times 1000}{(2 \times 10^{-1})}$$

$$= 1.66 \times 10^3 \text{ poise}$$

7. $R = n^{1/3} r \Rightarrow R = (64)^{1/3} r = 4r$

$$v_T \propto r^2 \Rightarrow \frac{v'_T}{1.5} = \left(\frac{R}{r} \right)^2 = 16$$

$$v'_T = 16 \times 1.5 = 24 \text{ ms}^{-1}$$

8. $F = \eta A \frac{v}{\ell} = 0.07 \times 0.1 \times \frac{1}{10^{-3}} = 7 \text{ N}$

9. Rate of flow $= Av = \pi r^2 \times \sqrt{2gH}$

$$= 3.14 \times (1)^2 \times \sqrt{2 \times 980 \times 10}$$

$$= 439.6 \approx 440 \text{ cc/sec}$$

10. $v_T = \frac{2}{9} \frac{r^2 (\rho_B - \rho_L) g}{\eta}$

$$v_T \propto \frac{r^2 g}{\eta}$$

$$\text{but } m \propto r^3 \Rightarrow \frac{m}{r} \propto r^2 \Rightarrow v_T \propto \frac{mg}{\eta r}$$

11. $v_T \propto (\rho_B - \rho_L)$

$$\frac{(v_T)_{\text{marble}}}{(v_T)_{\text{brass}}} = \frac{(\rho_m - \rho_L)}{(\rho_B - \rho_L)} = \frac{(2.5 - 0.8)}{(8.5 - 0.8)}$$

$$= \frac{1.7}{7.7} = \frac{17}{77}$$

12. $\eta = \frac{\text{shear stress}}{v/\ell}$

$$\text{Shear stress} = \eta \frac{v}{\ell} = 10^{-3} \times \frac{5}{10}$$

$$= 0.5 \times 10^{-3} \text{ N/m}^2$$

13. $v_T = \frac{2}{9} \frac{r^2 (\rho_B - \rho_L) g}{\eta}$

Here velocity are same

$$r^2 \propto \frac{1}{\rho_B - \rho_L}$$

$$\frac{r_A}{r_B} = \sqrt{\frac{(11 \times 10^3) - (2 \times 10^3)}{(8 \times 10^3) - (2 \times 10^3)}} = \sqrt{\frac{9}{6}} = \sqrt{\frac{3}{2}}$$

14. Rate of flow $\frac{dV}{dt} = \frac{\pi Pr^4}{8\eta \ell}$

$$Q = \frac{dV}{dt} \propto Pr^4$$

$$\frac{Q_2}{Q_1} = \left(\frac{P_2}{P_1} \right) \left(\frac{r_2}{r_1} \right)^4 = 2 \times \left(\frac{1}{2} \right)^4 = \frac{1}{8}$$

$$Q_2 = \frac{Q}{8}$$

15. $F_v = \eta A \frac{v}{\ell}$

$$(10^{-2} \times 10^5) = \eta \times 10^3 \times \frac{6}{0.6}$$

$$\eta = 0.1 \text{ poise}$$

Properties of Matter & Fluid Mechanics (Hydrostatics): RACE # 03

SOLUTION

1. Pressure due to liquid = $\frac{h}{2}\rho_w g + \frac{h}{2}\rho_\ell g$

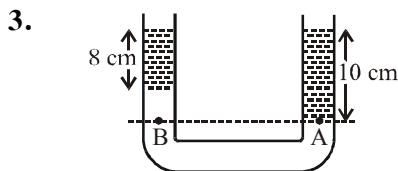
$$= \frac{h}{2}g(\rho_w + \rho_\ell)$$

$$= \frac{500}{2} \times g(1 + 0.85) = 462.5 \text{ g } \frac{\text{dyne}}{\text{cm}^2}$$

$$= 462.5 \frac{\text{g-wt}}{\text{cm}^2}$$

2. Volume of block inside the water
= $V - 0.1V = 0.9V$

For floating body $W = Th = V_{in} \rho_w g$
 $V \rho_B g = 0.9V \times 1g$
 $\rho_B = 0.9 \text{ g/cm}^3$



$$P_A = P_B$$

$$10 \times \rho_x \times g = 8 \rho_y g + 2 \rho_{Hg} g$$

$$10 \times 3.36 = 8 \rho_y + 2 \times 13.6$$

$$\rho_y = 0.8 \text{ g/cm}^3$$

4. For floating body $W = Th = V_{in} \rho_L g$
 $Ah \rho_B g = (Ah) \times 3 \rho_B g$
 $\frac{h}{H} = \frac{1}{3}$

$$\text{Exposed height ratio} = \frac{H-h}{H} = \frac{3-1}{3} = \frac{2}{3}$$

5. For floating body $W = Th = V_{in} \rho_w g$

$$Mg = (Ah) \times \rho_w g$$

$$21.6 \text{ g} = (0.3 \times 0.3)h \times 10^3 g$$

$$h = 24 \text{ cm}$$

6. By law of floating

$$Mg = \rho_1 V_1 g + \rho_2 V_2 g \Rightarrow M = \rho_1 V_1 + \rho_2 V_2$$

$$M = 0.6 \times 10^2 \times 4 + 0.4 \times 10^2 \times 6 = 480 \text{ gm}$$

Hence option (3)

7. $p = P_0 + \rho gh \Rightarrow p > P_0$
Hence option (2)

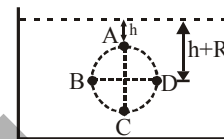
8. According to pascal's law pressure at same level is same.

$$\text{so } P_D = P_B = P_O + \rho g (h+R) \quad \dots(1)$$

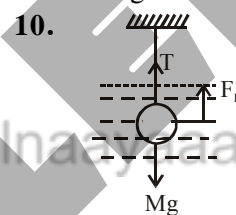
$$P_A = P_O + \rho gh \quad \dots(2)$$

$$P_C = P_O + \rho g (h+2R) \quad \dots(3)$$

$$\text{From eq. (1), (2) \& (3) } P_D = P_B = \frac{P_C + P_A}{2}$$



9. For floating $mg = \rho V_{in} g$
 $\Rightarrow V_{in} = \frac{m}{\rho}$ since inside volume is independent of 'g' so fraction of volume will remain same.



so

$$T = Mg - F_B$$

$$= \sigma Vg - \rho Vg$$

$$T = (\sigma - \rho) Vg \quad \dots(1)$$

$$Mg = \sigma Vg \quad \dots(2)$$

from eq. (1) & (2)

$$\Rightarrow \frac{T}{Mg} = \frac{\sigma - \rho}{\sigma} = \left(1 - \frac{\rho}{\sigma}\right)$$

$$T = Mg \left(1 - \frac{\rho}{\sigma}\right)$$

$$T = 3g \left(1 - \frac{0.8}{10}\right) = 30(0.92)$$

$$T = 27.6 \text{ N}$$

11. Suppose, l = side of the cube

$$x$$
 = side of cube immersed in water

$$l - x$$
 = side of cube immersed in liquid

According to law of floatation,

$$l^3 \times 0.9 \times 10^3 \times g = (l^2 \times x) \times 1000 g + l^2(l - x) \times 0.7 \times 10^3 g$$

$$l \times 0.9 = x + (l - x) \times 0.7$$

$$\text{or } 0.3x = 0.2l \quad \text{or} \quad \frac{x}{l} = \frac{2}{3}$$

12. $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$$F_2 = \left(\frac{A_2}{A_1}\right) F_1 = \left(\frac{R_2}{R_1}\right)^2 F_1$$

$$= \left(\frac{15}{3}\right)^2 \times 12 = 300 \text{ N}$$

13. $P_A = P_B$

$$h_w \rho_w g = h_{Hg} \rho_{Hg} g$$

$$11.2 \times 1 \times g = (2x) \times 13.6 \times g$$

$$x = \frac{11.2}{2 \times 13.6} = 0.41 \text{ cm}$$

14. Weight of the body = Weight of the liquid displaced

$$\text{In water } V \rho_{\text{block}} g = \left(\frac{4}{5} V\right) \rho_{\text{water}} g$$

$$\rho_{\text{block}} = \frac{4}{5} \rho_{\text{water}} \quad \dots (i)$$

In liquid,

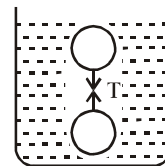
$$V \rho_{\text{block}} g = V \rho_{\text{liquid}} g$$

$$\rho_{\text{block}} = \rho_{\text{liquid}} \quad \dots (ii)$$

From equation (i) and (ii), we get

$$\begin{aligned} \rho_{\text{liquid}} &= \frac{4}{5} \rho_{\text{water}} = \frac{4}{5} \times 10^3 \text{ kg m}^{-3} \\ &= 800 \text{ kg m}^{-3}. \end{aligned}$$

15. The tension on heavier sphere is upwards and on lighter sphere is downwards



For lighter sphere $W + T = T_h$

$$250 \times 10^{-6} \times 800g + T = 250 \times 10^{-6} \times \rho_L \times g$$

For heavier sphere $W = T + T_h$

$$250 \times 10^{-6} \times 1200g = T + 250 \times 10^{-6} \rho_L g$$

on solving $T = 0.5 \text{ N}$

Properties of Matter & Fluid Mechanics (Surface Tension): RACE # 04

SOLUTION

1. Area of tube = πr^2

$$\Rightarrow \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \left(\frac{r_2}{r_1}\right) = \left(\frac{A}{4A}\right)^{\frac{1}{2}} = \frac{1}{2}$$

$$h \propto \frac{1}{r} \Rightarrow \frac{h_2}{10\text{cm}} = \frac{r_1}{r_2} = 2$$

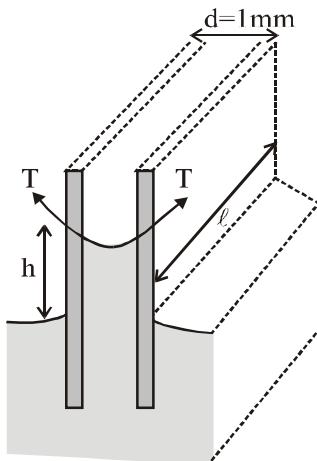
$$\Rightarrow h_2 = 20\text{ cm}$$

2. $2T\ell = mg \Rightarrow \ell = \frac{mg}{2T} = \frac{1 \times 980}{2 \times 70} = 7\text{ cm}$

3. Weight of drop $W = Mg = 2\pi rT$

$$\frac{W_1}{W_2} = \frac{T_1}{T_2} = \frac{30}{60} = \frac{1}{2}$$

4. $d = 1\text{ mm}$



Force upward = $(2T \cos \theta) \ell$ $\because (\theta = 0^\circ)$

Gravitational pull = $(\text{volume} \times \text{density})g = (\ell dh)\rho g$

Now upward force = Gravitational pull

$$\therefore (2T \cos 0^\circ) \ell = (\ell dh) \rho g$$

$$\frac{2T}{d\rho g} = h, \quad h = \frac{2 \times 70 \times 1}{0.1 \times 1 \times 980}$$

$$\Rightarrow h = 1.43\text{ cm}$$

5. $R = (n)^{\frac{1}{3}} \cdot r = 2r$

$$P_{\text{ex}} \propto \frac{1}{r} \Rightarrow \frac{(P_{\text{ex}})_{\text{big}}}{(P_{\text{ex}})_{\text{small}}} = \frac{r}{R} = \frac{1}{2}$$

$$\Rightarrow (P_{\text{ex}})_{\text{big}} = 3\text{ units}$$

6. $2T\ell = Mg \Rightarrow M = \frac{2T\ell}{g}$

$$= \frac{2 \times 3 \times 10^{-2} \times (10 \times 10^{-2})}{10} = 0.6\text{ g}$$

7. (a) $r = \frac{r_1 r_2}{r_1 - r_2} = \frac{80 \times 50}{80 - 50} = 133.3\text{ mm}$

(b) Shape will be concave.

8. $hR = h \frac{r}{\cos \theta} = \text{constant}$

$$\Rightarrow \frac{h}{\cos \theta} = \text{constant}$$

$$\frac{2}{\cos 0^\circ} = \frac{1}{\cos \theta} \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

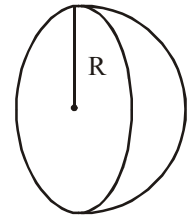
9. Surface tension = T

force due to surface

tension will be

$$= 2(2\pi R)T$$

$$= 4\pi RT$$



10. P.E. = Surface energy = $T \times 2\Delta A$

$$= 40 \times 10^{-3} \times 2 \times 4 \times 10^{-3}$$

$$= 320 \times 10^{-6} = 32 \times 10^{-5}\text{ J}$$

11. Let initial surface area = A

then final surface area = $\frac{A}{2}$

$$\% \text{ change in energy} = \frac{(SE)_f - SE_i}{SE_i} \times 100$$

$$= \frac{T \times 2\left(\frac{A}{2}\right) - T \times 2(A)}{T \times 2(A)} = -50\%$$

12. $(P_{\text{ex}})_1 = 3(P_{\text{ex}})_2 \Rightarrow \frac{4T}{r_1} = 3 \frac{4T}{r_2} \quad \frac{r_1}{r_2} = \frac{1}{3}$

Surface area = $4\pi r^2$

$$\Rightarrow \frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{9}$$

13. Circumference of flat plate = $L = 2\pi R = 10\pi\text{ cm}$.

$$\text{Hence, } T = \frac{F}{L}$$

$$\text{or } F = TL = 75 \times 10\pi = 750\pi\text{ dyne.}$$

14. $P_{\text{ex}} = \frac{4T}{r} = \frac{4 \times 1.6}{(2.5 \times 10^{-3})} = 2560 \text{ Pa}$

15. In case of soap bubble,
 $W = T \times 2 \times \Delta A$
 $= 0.03 \times 2 \times 40 \times 10^{-4}$
 $= 2.4 \times 10^{-4} \text{ J}.$

16. $F = T.2l$

or $T = \frac{F}{2l} = \frac{2 \times 10^{-2} \text{ N}}{2 \times (10 \times 10^{-2} \text{ m})}$
 $= 10^{-1} \text{ N/m} = 0.1 \text{ N/m}.$

17. Since, bubble has two surfaces,
 Initial surface area of the bubble
 $= 2 \times 4\pi r_1^2 = 2 \times 4\pi \times (3 \times 10^{-2})^2$
 $= 72\pi \times 10^{-4} \text{ m}^2$
 Final surface area of the bubble
 $= 2 \times 4\pi r_2^2 = 2 \times 4\pi (5 \times 10^{-2})^2 = 200\pi \times 10^{-4} \text{ m}^2$
 Increase in surface area
 $= 200\pi \times 10^{-4} - 72\pi \times 10^{-4} = 128\pi \times 10^{-4}$
 $\therefore \text{Work done} = T \times \text{increase in surface area}$
 $\approx 0.03 \times 128 \times \pi \times 10^{-4} = 3.84\pi \times 10^{-4}$
 $= 4\pi \times 10^{-4} \text{ J} = 0.4 \pi \text{ mJ}.$

18. Let r be the radius of each small droplets.
 Then, volume of big drop = $64 \times$ volume of each small droplets

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

$$R = 4r \quad \dots(i)$$

Surface area of big drop = $4\pi R^2$

Surface area of 64 small droplets = $64 \times 4\pi r^2$

$\therefore$ Increase in surface area = $64 \times 4\pi r^2 - 4\pi R^2$
 $= 4\pi[64r^2 - R^2]$
 $= 4\pi[4R^2 - R^2] \text{ [Using eqn.(i)]}$
 $= 12\pi R^2$

Energy needed = surface tension $\times$ Increase in surface area

$$= T \times 12\pi R^2 = 12\pi R^2 T$$

1. $\frac{F-32}{9} = \frac{C}{5}$

when $F = 2C \Rightarrow C = \frac{F}{2}$

therefore $\frac{F-32}{9} = \frac{F}{2(5)} \Rightarrow F = 320^\circ$

2. Thermal expansion of isotropic solid is similar to photographic enlargement so both r and R will increase.

3. If the sheet is heated then both d_1 and d_2 will increase since the thermal expansion of isotropic solid is similar to photographic enlargement.

4. Volume of water is minimum at 4°C it will overflow if both heated and cooled above & below 4°C respectively.

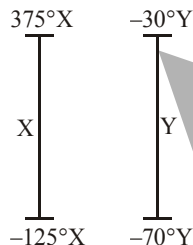
5. $x = \ell'_A - \ell'_B = \text{const}$ at any temperature

it means $\Delta\ell_A = \Delta\ell_B$

$\Rightarrow \ell_A \alpha_A \Delta T = \ell_B \alpha_B \Delta T$

$\Rightarrow \ell_A \alpha_A = \ell_B \alpha_B$

6. Let θ = temperature on X-scale corresponding to 50° on Y-scale



$\Rightarrow \frac{\theta - (-125)}{375 - (-125)} = \frac{50 - (-70)}{-30 - (-70)} \Rightarrow \theta = 1375^\circ\text{X}$

7. $\frac{T_K - 273}{100} = \frac{T_F - 32}{180} = \frac{T_C}{100} = \frac{30}{100}$

$\frac{T_K - 273}{100} = \frac{30}{100}$; $\frac{T_F - 32}{180} = \frac{30}{100}$

$(T_K = 303\text{K})$

$T_F = 54 + 32 = 86^\circ\text{F}$

8. $L_{100^\circ} = 60\text{ cm}$

$L_{0^\circ} = 10\text{ cm}$

$\frac{50-10}{60-10} = \frac{T-0}{100-0}$
 $T = 80^\circ$

9. $\frac{T_X - (20^\circ\text{X})}{220^\circ\text{X} - 20^\circ\text{X}} = \frac{T_Y - (-40^\circ\text{Y})}{120^\circ\text{Y} - (-40^\circ\text{Y})}$

$\frac{T_X - 20^\circ\text{X}}{200^\circ\text{X}} = \frac{T_Y + 40^\circ\text{Y}}{160^\circ\text{Y}}$

$\frac{100^\circ\text{X} - 20^\circ\text{X}}{200^\circ\text{X}} = \frac{T_Y + 40^\circ\text{Y}}{160^\circ\text{Y}}$

$\frac{80^\circ\text{X}}{200^\circ\text{X}} \times 160^\circ\text{Y} = T_Y + 40^\circ\text{Y}$

$T_Y = 24^\circ\text{Y}$

10. For A: upper pt. = 180
lower pt. = 30

For B: upper pt. = 100
lower pt. = 0

$\frac{t_A - 30}{180 - 30} = \frac{t_B - 0}{100 - 0}$

$\frac{t_A - 30}{150} = \frac{t_B}{100}$

11. $\Delta L = L \alpha \Delta T$

slope $\Rightarrow \frac{\Delta L}{\Delta T} = L\alpha$

$(\text{Slope})_A < (\text{Slope})_B$; $L_A \alpha_A < L_B \alpha_B$

$\therefore L_A = L_B$ therefore $\alpha_A < \alpha_B$

12. $L_1 < L_2$

B will expand same in both rods, Hence if $L_1 < L_2$ then A will expand less as compared to C
therefore $\alpha_A < \alpha_C$.

13. Liquid Overflows

$(\Delta V)_{\text{liq.}} > (\Delta V)_{\text{container}}$ ($\gamma_{\text{container}} = 3\alpha$)

$(V\gamma_\ell \Delta T) > V\gamma_c \Delta T$

$\gamma_\ell > \gamma_c$

$\gamma > 3\alpha$

14. $\gamma_{\text{apparent}} = \gamma_{\text{actual}} - \gamma_{\text{container}}$

For Copper Vessel : $C = \gamma - 3A$

For Silver Vessel : $S = \gamma - 3x$

$x = \frac{C + 3A - S}{3}$

Thermal Physics (Calorimetry) : RACE # 02

SOLUTION

1. Let specific heats be S_A and S_B respectively.
so Heat loss by A = Heat gain by B
 $mS_A(30 - 26) = mS_B(26 - 20)$

$$\Rightarrow \frac{S_A}{S_B} = \frac{3}{2}$$

2. Conversion of ice into steam will be as follows :
Ice at $-12^\circ\text{C} \xrightarrow{T \text{ increase}} \text{Ice at } 0^\circ\text{C} \xrightarrow{T=\text{constant}} \text{water at } 0^\circ\text{C}$

↓ T increase
water at 100°C
↓ T = constant
steam at 100°C

3. Water equivalent $w = ms = 20 \times 0.2 = 4\text{g}$

Available Heat	Required Heat
100g water (80°C)	100g ice (0°C)
$ms\Delta\theta \downarrow = 8000 \text{ cal}$	$mL \downarrow = 8000 \text{ cal}$
100g water (0°C)	100g water (0°C)

Therefore final temperature will be 0°C

5. Heat liberated by water to attain 0°C
 $= 10 \times 1 \times 40 = 400 \text{ cal}$

$$\text{Amount of ice melted} = \frac{400}{80} = 5 \text{ g}$$

$$\therefore \text{Total amount of water} = 10 + 5 = 15 \text{ g}$$

6. $m_{\text{ice}}S_{\text{ice}} \times 15 + m_{\text{ice}} \times L + m_{\text{ice}}S_{\text{water}}T_f$
 $= m_{\text{water}}S_{\text{water}}(28 - T_f)$
so $T_f = 7^\circ\text{C}$

Available Heat	Required Heat
1g steam (100°C)	2g ice (0°C)
$mL \downarrow = 540 \text{ cal}$	$mL \downarrow = 160 \text{ cal}$
1g water (100°C)	2g water (0°C)
	$ms\Delta\theta \downarrow = 200 \text{ cal}$
	2g water (100°C)

As required heat is less than available heat so final temperature will be 100°C .

8. Heat loss = Heat gain
 $mL_v + mS_w(100 - 90) = 22S_w(90 - 20)$
 $m[540 + 10] = 22 \times 1 \times 70$

$$m = \frac{22 \times 70}{550} = 2.8 \text{ gm}$$

$$\text{Mass of water} = 22 + 2.8 = 24.8 \text{ gm}$$

9. (i) $\left(\frac{dQ}{dt}\right)_A = \left(\frac{dQ}{dt}\right)_B$

$$\frac{Q_A}{t_A} = \frac{Q_B}{t_B} \Rightarrow \frac{Q_A}{Q_B} = \frac{t_A}{t_B} = \frac{7-4}{6-2} = \frac{3}{4}$$

$$\text{Also } \frac{L_A}{L_B} = \frac{Q_A}{Q_B} = \frac{3}{4} \quad (\because Q = mL)$$

$$(ii) \frac{dQ}{dt} = mS \frac{d\theta}{dt}$$

$$S \propto \frac{1}{\left(\frac{d\theta}{dt}\right)}$$

$$\frac{S_A}{S_B} = \frac{60/2}{40/4} = \frac{30}{10} = 3$$

10. Heat absorbed = Heat loss
 $mS_{\text{ice}}[0 - (-20)] + mL_f + mS_w(10 - 0)$
 $= 200 \times S_w(25 - 10)$
 $m[0.5 \times 20 + 80 + 10] = 200 \times 1 \times 15$
 $m = 30 \text{ gm}$

11. $\frac{dQ}{dt} = ms \frac{dT}{dt}$

$$1000 \times \frac{80}{100} = 20 \times 4200 \times \frac{(35-10)}{t}$$

$$t = \frac{20 \times 4200 \times 25}{800 \times 60} \text{ min} = 43.75 \text{ min}$$

$$\approx 44 \text{ min}$$

12. Heat loss = Heat gain
 $m_B S_w(80 - 50) = m_A S_w(50 - 30)$

$$\frac{m_A}{m_B} = \frac{30}{20} = \frac{3}{2} \quad \dots(i)$$

$$m_A + m_B = 40 \quad \dots(ii)$$

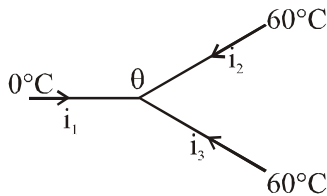
solving equation (i) & (ii) $m_A = 24 \text{ kg}$, $m_B = 16 \text{ kg}$

Thermal Physics (Modes of Heat Transfer) : RACE # 03

SOLUTION

1. Thermal resistance = $\frac{L}{KA}$ = same for all = R (let)

using kirchoff's law for heat current



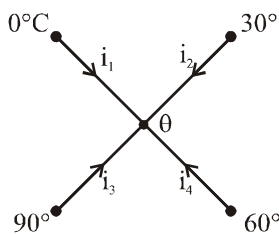
$$i_1 + i_2 + i_3 = 0$$

$$\Rightarrow \frac{0-\theta}{R} + \frac{60-\theta}{R} + \frac{60-\theta}{R} = 0$$

$$0 - \theta + 120 - 2\theta = 0 \Rightarrow 120 = 3\theta \Rightarrow \theta = 40^\circ\text{C}$$

2. Thermal resistance = $\frac{L}{KA}$ = same of each rod
= R (let)

$$\text{Heat current } i = \frac{dH}{dt}, \quad \frac{dH}{dt} = \frac{T_1 - T_2}{R}$$



using kirchoff's law for heat current,

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{(0-\theta)}{R} + \frac{(30-\theta)}{R} + \frac{(90-\theta)}{R} + \frac{(60-\theta)}{R} = 0$$

$$\Rightarrow 180 - 4\theta = 0 \Rightarrow \theta = 45^\circ\text{C}$$

3. The power radiated by a filament is $P = e\sigma AT^4$
where e = emissivity, σ = Stefan's constant,
 T = surface temperature. or $e_A T_A^4 = e_B T_B^4$

4. Temperature of body does not change. Hence energy radiated per second remains same.

$$5. \frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} = \frac{4800}{3600} = \frac{48}{36} = \frac{4}{3}$$

7. The frequency at which maximum intensity of emission occurs is proportional to T , while the rate of the emitted radiation is proportional to T^4 .

8. For $\theta - t$ plot, rate of cooling = $\frac{d\theta}{dt}$ = slope of the curve.

$$\text{At P, } \frac{d\theta}{dt} = |\tan(180 - \phi_2)| = \tan \phi_2 = k(\theta_2 - \theta_0)$$

where k = constant.

$$\text{At Q, } \frac{d\theta}{dt} = |\tan(180 - \phi_1)| = \tan \phi_1 = k(\theta_1 - \theta_0)$$

$$\therefore \frac{\tan \phi_2}{\tan \phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$$

9. As heat current is same in all cross section so option (1) is correct

$$10. \frac{Q}{t} = \frac{T_2 - T_1}{\frac{x}{KA} + \frac{4x}{2KA}} = \frac{KA(T_2 - T_1)}{3x}$$

$$\text{so } f = \frac{1}{3}$$

11. $Q = \sigma AT^4 t$
 $Q \propto R^2 T^4$

$$\frac{(Q)_1}{(Q)_2} = \frac{R^2}{10^4 R^2} \times \frac{T^4}{T^4} \times 16$$

$$Q_2 = \frac{10^4}{16} Q = 625Q$$

$$1. \quad P = (nRT) \frac{1}{V} \Rightarrow \frac{1}{V} = \frac{1}{nRT} P$$

$$\text{using } y = mx, \text{ we get, slope} = m = \frac{1}{nRT}$$

$$\text{slope} \propto \frac{1}{T} \text{ and } (\text{slope})_A > (\text{slope})_B > (\text{slope})_C$$

$$\Rightarrow T_A < T_B < T_C$$

$$2. \quad P \propto \frac{1}{T^2} \Rightarrow P = \frac{K}{T^2} \Rightarrow \frac{nRT}{V} = \frac{K}{T^2} \Rightarrow V \propto T^3$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{T_2^3}{T_1^3} \Rightarrow \frac{8V}{V} = \frac{T_2^3}{(300)^3}$$

$$\Rightarrow T_2^3 = 8(300)^3 \Rightarrow T_2^3 = [2 \times 300]^3$$

$$\Rightarrow T_2 = 600 \text{ K}$$

$$3. \quad V_{\text{RMS}} = \sqrt{\frac{1^2 + 2^2 + 3^2 + 4^2}{4}} = \sqrt{\frac{30}{4}} = \sqrt{\frac{15}{2}} \text{ km/s}$$

4. At very high temperature the H_2 gas molecule acquire two additional vibrational degree of freedom

$$\therefore \text{For a single } H_2 \text{ molecule, Total energy} = \frac{7}{2} kT$$

$$\text{Rotational energy} = \frac{2}{2} kT = kT$$

$$\therefore \text{Required ratio} = \frac{kT}{\frac{7}{2} kT} = \frac{2}{7}$$

5. At same temperature, RMS speed is independent of pressure.

$$6. \quad \frac{n_1 RT}{2V} = \frac{n_2 R 2T}{V} \Rightarrow n_2 = \frac{n_1}{4} \Rightarrow m_2 = \frac{m}{4}$$

$$7. \quad P = \frac{P_0}{1 + \left(\frac{V_0}{V}\right)^3} = \frac{P_0}{2} \Rightarrow T = \frac{P_0 V_0}{2R}$$

$\therefore$ Translational kinetic energy

$$= \frac{3}{2} RT = \frac{3R}{2} \frac{P_0 V_0}{2R} = \frac{3P_0 V_0}{4}$$

8. For same isotherm : $T \rightarrow \text{constant}$

$$\therefore P \propto \frac{1}{V} \Rightarrow P_1 V_1 = P_2 V_2$$

9. Option (1)

10. $PV \propto T$

$$\frac{PV}{T} = \text{Constant} = nR \Rightarrow \frac{PV}{T} = nR \Rightarrow \text{slope}$$

$$\text{No. of moles} = \frac{\text{Mass of gas}}{\text{Molecular weight}}$$

$$\frac{PV}{T} \propto n \propto \frac{1}{Mw} \quad \left(\text{Slope} \propto \frac{1}{Mw} \right)$$

$$(\text{slope})_B > (\text{slope})_A > (\text{slope})_C$$

$$\Rightarrow (Mw)_B < (Mw)_A < (Mw)_C$$

$$11. \quad \lambda = \frac{1}{\sqrt{2}\pi d^2 n} = \frac{KT}{\sqrt{2}\pi d^2 P} \quad (\because P = nKT)$$

n = Molecular density

$$\lambda = \frac{(1.38 \times 10^{-23}) \times 293}{\sqrt{2} \times \pi (3.1 \times 10^{-10})^2 \times (1.01 \times 10^5)}$$

$$(\lambda = 93.6 \text{ nm})$$

Thermal Physics (Thermodynamic process) : RACE # 05

SOLUTION

1. Process AB : $P = \text{constant} \Rightarrow V \propto T \Rightarrow \text{as } V \uparrow \text{ so } T \uparrow$
 Process BD : $V = \text{constant} \Rightarrow P \propto T \Rightarrow \text{as } P \downarrow \text{ so } T \downarrow$
 Process DC : $P = \text{constant} \Rightarrow V \propto T \Rightarrow \text{as } V \downarrow \text{ so } T \downarrow$
 Process CA : $V = \text{constant} \Rightarrow P \propto T \Rightarrow \text{as } P \uparrow \text{ so } T \uparrow$
 As we go from D to A, temperature first decrease then increase.

2. Let source and sink In Ist case, temperature be T_1 K and T_2 K

$$\text{efficiency } \eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{1}{2} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 1 - \frac{1}{2} \Rightarrow \frac{T_2}{T_1} = \frac{1}{2} \quad \dots(1)$$

In IInd case, efficiency, $\eta = 1 - \frac{(T_2 - 100)}{T_1}$

$$\Rightarrow \frac{2}{3} = 1 - \frac{T_2}{T_1} + \frac{100}{T_1} \Rightarrow \frac{2}{3} = 1 - \frac{1}{2} + \frac{100}{T_1} \text{ (using 1)}$$

$$\Rightarrow \frac{100}{T_1} = \frac{1}{6} \Rightarrow T_1 = 600 \text{ K}$$

3. COP refrigerator $\beta = \frac{T_2}{T_1 - T_2} = \frac{250}{50} = 5$ Also,

$$\beta = \frac{Q_2}{W} \Rightarrow 5 = \frac{Q_2}{W} \Rightarrow 5 = \frac{Q_2}{(1)}$$

$$Q_2 = 5 \text{ J}$$

Heat delivered to surroundings $= Q_1 = Q_2 + W$
 $= 5 + 1 = 6 \text{ J}$

4. $\Delta U = nC_V \Delta T = nC_V (T_2 - T_1) = \frac{nR}{\gamma - 1} (T_2 - T_1)$

$$= \frac{nRT_2 - nRT_1}{\gamma - 1} = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

5. At no exchange condition, i.e. adiabatic process, $Q = 0$

$$C = \frac{Q}{n\Delta T} = \frac{0}{n\Delta T} = 0$$

for isothermal process, $C = \frac{Q}{n(0)} = \infty (\because \Delta T = 0)$

At constant pressure, $C = C_P = \frac{R\gamma}{\gamma - 1}$

At constant volume, $C = C_V = \frac{R}{\gamma - 1}$

6. Work done $= -$ (area of the closed loop)

$$= - \pi \times \left(\frac{3P_0 - P_0}{2} \right) \times \left(\frac{3V_0 - V_0}{2} \right)$$

$$= - \frac{22}{7} P_0 V_0$$

7. Process A : $T = \text{constant} \Rightarrow$ Isothermal process

Process B : $P = \text{constant} \Rightarrow$ isobaric process

Process C : $P \propto T \Rightarrow V = \text{constant} \Rightarrow$ isochoric

Process D : No well defined equation

8. From first law of thermodynamics

$$dQ = dU + dW$$

$$\Rightarrow nCdT = nC_V dT + PdV$$

$$\Rightarrow C = C_V + \frac{P}{n} \frac{dV}{dT}$$

Now from $PV = nRT$ & $PV^x = \text{constant}$,

We have $T \propto V^{1-x}$

$$\Rightarrow \frac{dT}{T} = (1-x) \frac{dV}{V} \Rightarrow \frac{dV}{dT} = \frac{V}{(1-x)T}$$

$$\text{therefore } C = C_V + \frac{P}{n} \frac{V}{(1-x)T} = C_V + \frac{R}{1-x}$$

9. Here $P = aV^2 \Rightarrow x = -2$, from previous question

$$\text{so } C = C_V + \frac{R}{1+2} = \frac{5}{2}R + \frac{R}{3} = \frac{17R}{6}$$

10. Efficiency of refrigerator $\beta = \frac{T_2}{T_1 - T_2}$

$$\beta = \frac{273}{300 - 273} = \frac{273}{27}$$

$$\text{Also, } \beta = \frac{Q_2}{W} \Rightarrow \frac{273}{27} = \frac{Q_2}{(1)} \Rightarrow Q_2 = \frac{273}{27}$$

$\therefore$ Heat delivered to surroundings

$$Q_1 = Q_2 + W = \frac{273}{27} + 1 = \frac{300}{27} = \frac{100}{9} = 11.11 \text{ J}$$

11. Loss in KE of box = gain in internal energy of gas

$$\Rightarrow \frac{1}{2}mv^2 = nC_v \Delta T$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{m}{M} \times \frac{R}{\gamma - 1} \Delta T$$

$$\Rightarrow \Delta T = \frac{(\gamma - 1)}{2R} Mv^2$$

12. $PV^\gamma = k$

$$\ln PV^\gamma = \ln k \Rightarrow \ln P + \gamma \ln V = \ln k$$

$$\Rightarrow \ln P = -\gamma \ln V + \ln k$$

Compare with $y = mx + c$

$$m = -\gamma$$

Line with more negative slope has more γ

13. The change in the internal energy = $nC_v \Delta T$

14. For small change $dQ = dU + dW$;

$$nC_v dT = nC_v dT + 2RdT$$

$$\therefore C = C_v + 2R$$

$$\therefore C_v = 2R$$

$$\text{so } C_p = C_v + R = 3R$$

$$\text{therefore } \gamma = \frac{C_p}{C_v} = \frac{3}{2}$$

15. $(W)_{ABC} = \frac{1}{2} \times (4-2) \times (2-1) \Rightarrow +1J$ (Clockwise)

$$(W)_{ADE} = \frac{1}{2} \times (2-1) \times (3-2) = -\frac{1}{2}J \text{ (Anticlockwise)}$$

$$\left(\text{Net Work} = +\frac{1}{2}J \right)$$

16. $Q = \Delta U + W$

$$\Delta U = nC_v \Delta T = 1 \times \frac{3}{2}R \times (2T_0 - T_0) = \frac{3}{2}RT_0$$

$$\text{Work} = \frac{nR\Delta T}{1-x} = \frac{1 \times R \times (2T_0 - T_0)}{2} \Rightarrow \left(\frac{RT_0}{2} \right)$$

$$[\because x = -1]$$

$$Q = \frac{3}{2}RT_0 + \frac{RT_0}{2} = 2RT_0$$

17. Case-1: $T_2 = 0^\circ\text{C} = 273K$

$$T_1 = 200^\circ\text{C} = 473K$$

$$\eta_1 = 1 - \frac{273}{473} = \frac{200}{473}$$

- Case-2: $T_2' = -200^\circ\text{C} = 73K$

$$T_1' = 0^\circ\text{C} = 273K$$

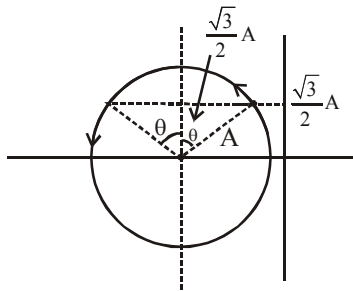
$$\eta_2 = 1 - \frac{73}{273} = \eta_2 = \frac{200}{273}$$

$$\frac{\eta_2}{\eta_1} = \frac{473}{273} = \frac{1.73}{1}$$

Oscillations (Kinematics of SHM) : RACE # 01

SOLUTION

1.



$$\cos \theta = \frac{\sqrt{3}A/2}{A} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

Phase difference = $2\theta = 60^\circ$

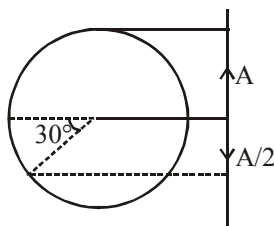
OR

$$x_1 = A \sin \omega t \text{ \& } x_2 = A \sin (\omega t + \phi)$$

$$\text{Here } x_1 = x_2 = \frac{\sqrt{3}A}{2}$$

$$\Rightarrow \omega t = \frac{\pi}{3} \text{ \& } \omega t + \phi = \frac{2\pi}{3} \Rightarrow \phi = \frac{\pi}{3}$$

2. $\frac{5}{8} (4A) = 2.5 A = 2A + 0.5 A$



total angle travelled

$$= 180^\circ + 30^\circ = 210^\circ \times \frac{\pi}{180} = \frac{7\pi}{6}$$

$$t = \frac{7\pi}{6} \times \frac{T}{2\pi} = \frac{7}{12} T$$

OR

$$\therefore \frac{5}{8} \text{ oscillation} = \frac{1}{2} \text{ oscillation} + \frac{1}{8} \text{ oscillation}$$

(from mean position)

$$\therefore \text{Time taken} = \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$$

3. $\frac{d^2x}{dt^2} + \frac{320}{4}x = 0$

$$\frac{d^2x}{dt^2} + 80x = 0$$

$$\text{so } \omega^2 = 80$$

$$\omega = \sqrt{80} = 4\sqrt{5}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\sqrt{5}} = \frac{\pi}{2\sqrt{5}} \text{ s}$$

4. $v_{\max} = A\omega = 40 \text{ cm/s}$

$$\text{Here } A = 10 \text{ so } \omega = 4$$

$$T = \frac{2\pi}{4} = \frac{\pi}{2} \text{ s}$$

5. From diagram

amplitude = 4 cm, Time period $T = 12 \text{ s}$

Let equation be $x = A \sin (\omega t + \phi)$

$$\text{At } t = 0, x = 2 \text{ so } 2 = 4 \sin (\phi) \Rightarrow \phi = \frac{\pi}{6}$$

$$\Rightarrow x = 4 \sin \left(\frac{2\pi}{12} t + \frac{\pi}{6} \right)$$

6. $v = -10 (2\pi) \sin (2\pi t + \frac{\pi}{2})$

$$\text{At } t = \frac{1}{6}$$

$$v = -20\pi \sin \left(\frac{\pi}{3} + 90^\circ \right) = -20\pi \cos \frac{\pi}{3}$$

$$\text{Magnitude of velocity} = 20\pi \times \frac{1}{2} = 10\pi$$

$$\Rightarrow v = 31.4 \text{ cm/s}$$

7. $F = -\frac{dU}{dx} = -(0 + 4\beta x)$

$$ma = -4\beta x \Rightarrow a = -\frac{4\beta}{m}x = -\omega^2 x$$

$$T = 2\pi \sqrt{\frac{m}{4\beta}} \Rightarrow$$

$$T = \pi \sqrt{\frac{m}{\beta}}$$

8. (a) $T = \frac{2\pi}{\omega} = \frac{2\pi}{6.28} = 1 \text{ s}$

(b) $v_{\max} = A\omega = (10 \text{ cm}) (6.28)$
 $= 62.8 \text{ cm/s} = 0.628 \text{ m/s}$

(c) $a_{\max} = \omega^2 A = (6.28)^2 (0.1) = 4 \text{ m/s}^2$

(d) $v = \omega \sqrt{A^2 - x^2}$
 $= 6.28 \sqrt{100 - 36} = 8 \times 6.28 = 50.2 \text{ cm/s}$

(e) $v = -\omega A \sin \omega t$

$$= -\omega A \sin \left(\frac{2\pi}{6} \right) = -54.4 \text{ cm/s}$$

9. $\theta = \theta_0 \sin(\omega t + 90^\circ)$

$$\theta = \frac{\pi}{10} \cos\left(\frac{2\pi}{0.05}t\right)$$

$$\theta = \frac{\pi}{10} \cos(40\pi t)$$

10. $\frac{d^2x}{dt^2} = -4x \Rightarrow \omega^2 = 4 \quad \text{or} \quad \omega = 2$

$A = 1\text{m}$. Maximum speed $= A\omega = 2\text{m/s}$

Maximum acceleration $= -\omega^2 A = -4(1)$
 $= -4 \text{ m/s}^2$

11. at $t = 0.5 \text{ sec}$

$\phi_1 = (5\pi + \pi/3)$ & $\phi_2 = (4\pi + \pi/4)$

$\Delta\phi = \phi_1 - \phi_2 = 195^\circ$

12. $y = a \sin \omega t$

$$= a \sin\left(\frac{2\pi}{T} \times \frac{T}{8}\right) = \frac{a}{\sqrt{2}}$$

13. $a \propto -x$

$\therefore \Delta\phi = \pi$

14. The equation $a = -100x + 50$ ($a = -kx$ form) itself shows that the particle performs SHM.

Hence (4)

15. Given $\omega^2 A = 35.28$

$\omega = 4.2$

$\ast (4.2)^2 A = 35.28$

Oscillations (Energy & Spring Pendulum) : RACE # 02

SOLUTION

1. Total mechanical energy always remains constant.

$$\text{So } \omega = 0, T = \frac{2\pi}{\omega} = \text{Infinite}$$

2. $\frac{1}{2}KA^2 = \frac{1}{2}K(0.2)^2 = 1$

$$K = \frac{2}{(0.2)^2} = \frac{2 \times 100}{4} = 50$$

$$\omega = \sqrt{K/m}$$

$$= \sqrt{\frac{50}{4}} = \frac{5}{2}\sqrt{2} = \frac{5}{\sqrt{2}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\sqrt{2}\pi}{5} \text{ s}$$

3. $T = 2\pi \sqrt{\frac{m}{k}}$

$$4 = 2\pi \sqrt{\frac{m}{k}} \quad \dots(1)$$

$$5 = 2\pi \sqrt{\frac{m+2}{k}} \quad \dots(2)$$

so eqⁿ (2)/(1)

$$\frac{5}{4} = \sqrt{\left(\frac{m+2}{k}\right) \frac{k}{m}}$$

$$\Rightarrow \frac{25}{16} = \frac{m+2}{m} \Rightarrow m = 3.5 \text{ kg}$$

4. By F-x graph

$$\frac{dF}{dx} = -10$$

$$F = -10x \Rightarrow a = -\frac{10}{m}x$$

$$\omega^2 = \frac{10}{m} \quad \dots(1)$$

By x-t graph

$$T = 8\text{s}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4} \Rightarrow \omega = \frac{\pi}{4} \text{ s}^{-1}$$

$$\text{from (1)} \quad m = \frac{10}{\omega^2} = \frac{10}{\pi^2} \times 16 = \frac{160}{\pi^2}$$

$$(K.E.)_{\max} = \frac{1}{2}kA^2$$

$$= \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} \cdot \frac{160}{\pi^2} \cdot \frac{\pi^2}{16} (4 \times 10^{-2})^2 = 8 \times 10^{-3}$$

5. $k = 2 \times 10^6 \text{ N/m}$, $A = 0.01\text{m}$, M.E. = 160 J

$$\text{Max K.E.} = \frac{1}{2}KA^2 = 100\text{J}$$

$$\text{Max P.E.} = 160$$

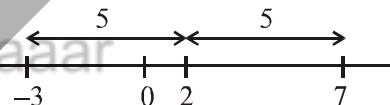
6. $U = 2 - 20x + 5x^2$

$$F = -\frac{dU}{dx} = 20 - 10x$$

At equilibrium position ; $F = 0$

$$20 - 10x = 0 \Rightarrow x = 2$$

Since particle is released at $x = -3$, therefore amplitude of particle is 5



It will oscillate about $x = 2$ with an amplitude of 5.

$\therefore$ maximum value of x will be 7

7. P.E is maximum at extreme position and minimum at mean position.

Time to go from extreme position to mean

position is, $t = \frac{T}{4}$; where T is time period of SHM

$$\therefore T = 4t = 20\text{s}$$

9. $V = \pm\omega\sqrt{A^2 - x^2}$, $PE = \frac{1}{2}kx^2$

$$a = -\omega^2x,$$

$$KE = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$$

Ratio of acceleration to displacement

$$= \frac{-\omega^2x}{x} = -\omega^2 \text{ (constant)}$$

10. $f = \frac{1}{2\pi} \sqrt{\frac{k_{eq}}{m}}$

$$\frac{f_1}{f_2} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2}$$

11. Spring constant $k = \frac{6.4}{0.1} = 64 \text{ N/m}$.

Now $T = 2\pi \sqrt{\frac{m}{k}}$ or $\frac{\pi}{4} = 2\pi \sqrt{\frac{m}{64}} \therefore m = 1 \text{ kg}$

$$f = \frac{1}{2\pi} \sqrt{\frac{k_{eq}}{M}} = \frac{1}{2\pi} \sqrt{\frac{4k}{M}}$$

12. Both the spring are in series

$$\therefore k_{eq} = \frac{k(2k)}{k+2k} = \frac{2k}{3}$$

Time period

$$T = 2\pi \sqrt{\frac{\mu}{k_{eq}}}$$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Here $\mu = \frac{m}{2} \therefore T = 2\pi \sqrt{\frac{\frac{m}{2} \cdot 3}{2k}} = 2\pi \sqrt{\frac{3m}{4k}}$

13. $Mg = kx_0$
 $(M + m)g = k(x_0 + x)$
 $Mg = kx$

$$T = 2\pi \sqrt{\frac{\text{mass}}{k_{eq}}}$$

$$\Rightarrow 2\pi \sqrt{\frac{(M+m)}{\frac{mg}{x}}}$$

$$= 2\pi \sqrt{\frac{(M+m)x}{mg}}$$

Oscillations (Simple pendulum and types of SHM) : RACE 03

SOLUTION

1. $nT_1 = (n+1)T_2$

$$\Rightarrow (n) 2\pi \sqrt{\frac{1.21}{g}} = (n+1) 2\pi \sqrt{\frac{1}{g}}$$

$$\Rightarrow \frac{11n}{10} = n+1 \Rightarrow 11n = 10n + 10 \Rightarrow n = 10$$

2. $T = 60s = 2\pi \sqrt{\frac{\ell}{g}}$

$$\ell' = 1.44 \ell$$

$$T' = 2\pi \sqrt{\frac{1.44\ell}{g}} = \sqrt{1.44} \times 2\pi \sqrt{\frac{\ell}{g}}$$

$$= \frac{12}{10} \times 60 = 72 \text{ sec.}$$

3. $mg = Kx$

$$K = \frac{mg}{x} = \frac{100 \times 10^{-3} \times 9.8}{9.8 \times 10^{-2}} = 10$$

$$T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2}$$

$$= \frac{6.28 \times 6.28 \times 10}{4\pi^2} = 10 \text{ kg} = 10^4 \text{ g}$$

5. (a) $T = 2\pi \sqrt{\frac{\ell}{g}}$

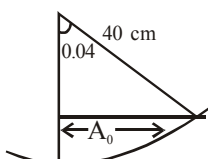
$$= 2\pi \sqrt{\frac{40 \times 10^{-2}}{10}}$$

$$= \frac{2\pi \times 2}{10} = \frac{4 \times 3.14}{10} = 1.26 \text{ s}$$

(b) $\sin(0.04 \times \frac{180}{\pi}) = \frac{A_0}{40}$

$$\Rightarrow A_0 = 40 \sin\left(0.04 \times \frac{180}{\pi}\right)$$

$$\Rightarrow A_0 = 1.6 \text{ cm}$$



(c) Angular velocity

$$= \omega \sqrt{\theta_0^2 - \theta^2}$$

$$= \frac{2\pi}{1.26} \sqrt{(0.04)^2 - (0.02)^2}$$

$$= \frac{2\pi}{1.26} \cdot \frac{\sqrt{12}}{100} = 0.172 \text{ rad/s}$$

linear speed = ℓ (angular velocity)

$$= 40 (0.172)$$

$$= 6.8 \text{ cm/s}$$

(d) $\alpha = \omega^2 \theta_0$

$$= \left(\frac{2\pi}{1.26}\right)^2 (0.04)$$

$$= 1 \text{ rad/s}^2$$

6. $T = 4 \text{ sec} \Rightarrow \omega = \frac{2\pi}{T} = \frac{\pi}{2} = 0.5\pi$

$$A = 0.2 \text{ m}$$

at $t = 0$, $x = 0.1 \text{ m} = \frac{A}{2}$

$$\therefore \Delta\phi = \pi/6$$

7. $f = \frac{1}{2\pi} \sqrt{\frac{g+1}{1}} \approx \frac{1}{2\pi} \times \pi = \frac{1}{2} \text{ Hz}$

8. Time taken by the pendulum to move from A

to B and from B to A is $\frac{T}{2}$.

Time period of oscillation $\propto \sqrt{L}$.

$$\therefore \frac{T_1}{T} = \sqrt{\frac{L/4}{L}} = \frac{1}{2} \text{ or } T_1 = \frac{T}{2}$$

Time taken to complete half the oscillation = $\frac{T}{4}$.

Total time period of oscillation

$$= \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

9. $T = 2\pi\sqrt{\frac{l}{g}}$

As initially centre of mass goes down and then comes to its original position, therefore, effective length of pendulum first increases and then decreases to the original value. So, will be the time period.

11. Let $T = 2\pi\sqrt{\frac{l}{g}}$

Let V be the volume of the mass

$V \times \rho \times g$ is acting downwards

$V \times \frac{\rho}{8} \times g$ is acting upwards

Inside the liquid, the effective force downwards

$$= V\rho g \times \frac{7}{8}, \text{ i.e., effective } g \text{ is } \frac{7}{8}g$$

$$\therefore T' = 2\pi\sqrt{\frac{l}{(7/8)g}} \text{ or } T' = 2\pi\sqrt{\frac{8l}{7g}} = \sqrt{\frac{8}{7}}T.$$

12. $T_p = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + s^2}}} < T$

$$T_s = 2\pi\sqrt{\frac{m}{k}} = T$$

1. In hydrogen gas,

$$V_{H_2} = \sqrt{\frac{\gamma_{H_2} RT}{m_{H_2}}} = \sqrt{\frac{7}{5} \frac{RT}{2}} = \sqrt{\frac{7}{10} \frac{RT}{1}} = v_0 \quad \dots(1)$$

Let mixture contains x moles each of H_2 & He

$$\gamma_{\text{mix}} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}}$$

$$\gamma_{\text{mix}} = \frac{x \times \frac{7R}{2} + x \times \frac{5R}{2}}{x \times \frac{5R}{2} + x \times \frac{3R}{2}} = \frac{3}{2}$$

$$m_{\text{mix}} = \frac{x \times 2 + x \times 4}{x + x} = 3$$

$\therefore$ velocity of sound in mixture,

$$v_{\text{mix}} = \sqrt{\frac{\gamma_{\text{mix}} RT}{m_{\text{mix}}}} = \sqrt{\frac{3}{2} \frac{RT}{3}}$$

$$v_{\text{mix}} = \sqrt{\frac{RT}{2}} \quad \dots(2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{v_0}{v_{\text{mix}}} = \frac{\sqrt{\frac{7}{10} \frac{RT}{1}}}{\sqrt{\frac{RT}{2}}} = \sqrt{\frac{14}{10}} = \sqrt{\frac{7}{5}}$$

$$v_{\text{mix}} = \sqrt{\frac{5}{7}} v_0 \quad \text{Hence option (1)}$$

2. Given $\omega = 3\pi$

$$\therefore f = \frac{\omega}{2\pi} = 1.5,$$

$$\text{Also } \Delta x = 1.0 \text{ cm}$$

$$\text{Given, } \phi = \frac{2\pi}{\lambda} \Delta x \Rightarrow \frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1$$

$$\Rightarrow \lambda = 16 \text{ cm} \Rightarrow v = f \lambda = 1.5 \times 16 = 24 \text{ cm/sec}$$

$$3. \quad \frac{\lambda}{2} = 10 \text{ m}, T = 4 \text{ s}, n = 1/4$$

$$\lambda = 20 \text{ m}$$

$$v = n\lambda = 1/4 \times 20 = 5 \text{ m/s}$$

$$4. \quad y = 3 \sin 6.28 (0.5 x - 50 t)$$

$$\omega = 6.28 \times 50 \quad k = 6.28 \times 0.5$$

$$v = \frac{\omega}{k} = 100 \text{ cm/sec}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{6.28 \times 0.5}$$

$$\Rightarrow \lambda = 2 \text{ cm}$$

$$5. \quad (a) 6 \text{ m}, 0.25 \text{ Hz}, 1.5 \text{ m/s}$$

$$(b) 1.5\pi \text{ mm/s}, 0.75\pi^2 \text{ mm/s}$$

6. Wave (1) & (3) having same frequency and (2) is less frequency.

7. Both points 'B' and 'G' are moving up and are at the same distance from equilibrium position at the instant shown.

$$8. \quad V = \sqrt{\frac{T}{\mu}} = \frac{\omega}{k} \quad \mu \rightarrow \text{linear mass density}$$

$$\Rightarrow T = \frac{\omega^2 \mu}{k^2} = \left(\frac{420}{21} \right)^2 \times 0.2 = 80 \text{ N.}$$

$$9. \quad \text{The speed of sound in air is } v = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{\gamma}{M} \text{ of } H_2 \text{ is largest, hence speed of sound in}$$

H_2 shall be maximum.

10. In a gas,

$$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T}$$

$$\Rightarrow \frac{v}{2v} = \sqrt{\frac{300}{T}} \Rightarrow T = 300 \times 4 = 1200 \text{ K}$$

$$= 927^\circ \text{C}$$

11. $n = 384 \text{ Hz}$, $T = \frac{1}{384} \text{ sec.}$

for 36 vibration time is, $T' = \frac{36}{384} = \frac{3}{32}$

so distance $d = v \times t = 352 \times 3/32 = 33\text{m}$

12. For a given medium speed of sound wave is constant so all three sound waves will travel with same speed.

13. $y = \cos (500 t - 70x)$

$$v = \frac{\omega}{k} = \frac{500}{70} = \frac{50}{7} \text{ m/s}$$

$$k = \frac{2\pi}{\lambda}, \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{70} \text{ m}$$

so $\lambda = \frac{20\pi}{7} \text{ cm}$

14. $f_1 \lambda_1 = f_2 \lambda_2$
 $(300) (1) = (f_2) (1.5)$
 $200 \text{ Hz} = f_2$

16. $v = \sqrt{\frac{\gamma P}{\rho_a}} = \sqrt{\frac{\gamma \times h \rho_m g}{\rho_a}}$
 $= \sqrt{\frac{1.41 \times 0.76 \times 13.6 \times 10^3 \times 9.8}{1.293}}$
 $= 332.3 \text{ m/s.}$

17. We know that light waves are electromagnetic in nature. Therefore, they donot require a medium for propagation. Thus, the light wave can travel in vacuum. On the other hand, sound waves require a medium for their propagation. They are mechanical waves and cannot travel in vacuum.

Wave Motion & Doppler's Effect : RACE # 02
(Superposition of waves interference, beats)

SOLUTION

1. $\therefore$ No. of beats = 6
2. Phase different between (1) & (3) wave is π rad
so its resultant = $10 - 7 = 3$
so resultant of 4 & 3 is 5 μm
3. $\frac{a_1 + a_2}{a_1 - a_2} = 5 \Rightarrow a_1 + a_2 = 5(a_1 - a_2)$

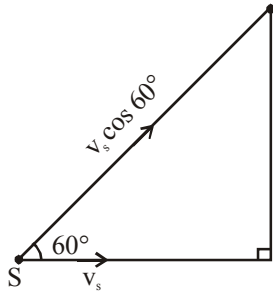
$$\frac{a_1}{a_2} = \frac{3}{2} \quad \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{9}{4}$$
4. $n_A = 256$
As tuning fork A when sounded with tuning fork B gives four beats, therefore the frequency n_B of tuning fork B is,
 $n_B = 256 \pm 4 = 252$ or 260
When the tuning fork A is slightly loaded with wax, the frequency of A decreases and the difference between two frequencies decreases if
 $n_B = 252$
As on sounding beats decrease, hence
 $n_B = 252$ Hz.
5. Due to different frequency intensity will vary with true. But it's value will change between max. and min.
6. Additional path difference introduction when the piston is moved through $8.75 \text{ cm} = 2 \times 8.75 = 17.50 \text{ cm}$
Since, the intensity changes from a maximum to a minimum additional path = $\lambda/2$
 $\therefore (\lambda/2) = 17.50$ or $\lambda = 35.0 \text{ cm}$
velocity, $v = 350 \text{ m/s} = 35000 \text{ cm/s}$
 $\therefore$ Frequency, $n = \frac{v}{\lambda} = \frac{35000}{35} = 1000 \text{ Hz}$.

7. $y_1 = a_1 \sin(\omega t - kx)$
 $y_2 = a_2 \sin(\omega t - kx + \phi + \frac{\pi}{2})$
 $\Delta\phi = \phi + \frac{\pi}{2}$
 $\Delta x = \left(\frac{\Delta\phi}{2\pi}\right)\lambda = \left(\frac{\phi + \frac{\pi}{2}}{2\pi}\right)\lambda$
8. Beat frequency = number of beats/sec,
 $n = n_2 \sim n_1$
 $\therefore n_1 = n_2 \pm n$
9. Path difference, $\Delta x = S_2 D - S_1 D$
 $= 5 - 4 = 1 \text{ m}$
 $\therefore$ The corresponding phase difference will be,
 $\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{4} \cdot 1 = \frac{\pi}{2}$
- Using $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$
 $I = I_0 + I_0 + 2\sqrt{I_0 I_0} \cos\frac{\pi}{2} = 2I_0$
10. $n \pm 5 = 440 \text{ Hz}$
and $n \pm 8 = 437 \text{ Hz}$
 $\therefore n = 445 \text{ Hz}$ (by both the methods)
Beat frequency condition will be satisfied with 445 Hz.
11. Resultant amplitude,
 $A_R = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$
 $= \sqrt{A^2 + A^2 + 2A^2 \cos\phi}$
But $A_R = A$
 $\therefore A = \sqrt{2A^2(1 + \cos\phi)}$
 $= \sqrt{4A^2 \cos^2 \frac{\phi}{2}} = 2A \cos \frac{\phi}{2}$
or $\cos \frac{\phi}{2} = \frac{1}{2}$ or $\phi = \frac{2\pi}{3}$

Wave Motion & Doppler's Effect : RACE # 03
(Stationary waves & doppler effect, beats)

SOLUTION

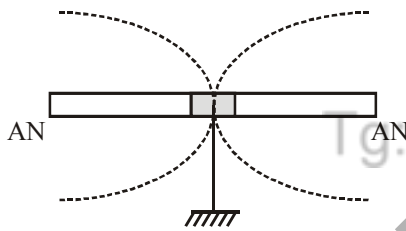
1.



frequency heard by observed

$$\begin{aligned} v' &= \left(\frac{v}{v - v_s \cos 60^\circ} \right) v \\ &= \left(\frac{330}{330 - 20 \times \frac{1}{2}} \right) 640 \\ &= \frac{330}{320} \times 640 = 660 \text{ Hz} \end{aligned}$$

2.



$$\begin{aligned} \text{Distance between two antinodes} &= \frac{\lambda}{2} = \text{length of rod} \\ &= \frac{\lambda}{2} = 1 \Rightarrow \lambda = 2\text{m} \end{aligned}$$

speed of sound in rod

$$= \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.8 \times 10^{10}}{2600}} = 5.47 \times 10^3 \text{ m/s}$$

$$\therefore \text{Frequency of sound, } v = \frac{v}{\lambda} = \frac{5.47 \times 10^3}{2} \approx 2740 \text{ Hz}$$

3. Let component waves are $y_1 = a \sin(kx - \omega t)$ & $y_2 = a \sin(kx + \omega t)$

$$\therefore \text{Resultant wave, } y = 2a \sin kx \cos \omega t$$

$$\Rightarrow \text{Amplitude} = |2a \sin kx|$$

$$\therefore \text{At antinode, amplitude} = 2a = 4$$

$$\therefore \text{At required point, amplitude} = \left| 2a \sin k \times \frac{\lambda}{8} \right|$$

$$= \left| 2a \sin \left[\frac{2\pi}{8} \right] \right|$$

$$\begin{aligned} &= \left| 2a \sin \frac{\pi}{4} \right| = 4 \times \frac{1}{\sqrt{2}} \\ &= 2\sqrt{2} \text{ cm} \end{aligned}$$

$$5. \quad f = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}}$$

If radius is doubled and length is doubled, mass per unit length will become four times. Hence

$$f' = \frac{1}{2 \times 2\ell} \sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$$

6. Equation of the component wave are $y = A \sin(\omega t - kx)$ and $y = A \sin(\omega t + kx)$ where ; $\omega t - kx = \text{constant}$ or $\omega t + kx = \text{constant}$ Differentiating w.r.t 't';

$$\omega - k \frac{dx}{dt} = 0 \quad \text{and} \quad \omega + k \frac{dx}{dt} = 0$$

$$\Rightarrow v = \frac{dx}{dt} = \frac{\omega}{k} \quad \text{and} \quad v = -\frac{\omega}{k}$$

i.e. ; the speed of component waves is $\left(\frac{\omega}{k} \right)$

$$7. \quad f_1 = \frac{1}{2\ell_1} \sqrt{\frac{T}{\rho \pi r_1^2}} \quad \text{and} \quad f_2 = \frac{1}{2\ell_2} \sqrt{\frac{T}{\rho \pi r_2^2}}$$

$$\Rightarrow f_1 = f_2$$

$$\Rightarrow \frac{1}{2\ell_1} \times \frac{1}{2} = \frac{1}{2\ell_2} \Rightarrow \ell_1 : \ell_2 = 1 : 2$$

$$8. \quad K = \frac{2\pi}{\lambda} = 0.025\pi \Rightarrow \frac{\lambda}{2} = \frac{1}{0.025} = 40\text{cm}$$

The shortest possible length will be

$$L_{\min} = \frac{\lambda}{2} = 40\text{cm}.$$

$$9. \quad \frac{5v}{2\ell} = 480\text{Hz} \Rightarrow \ell = \frac{5v}{960} \text{ m}$$

$$f = \frac{2v}{2\ell} = \frac{v}{\ell} = \frac{v \times 960}{5v} = 192 \text{ Hz}$$

$$10. f = \frac{4v}{2\ell} = \frac{2\sqrt{\frac{T}{\mu}}}{\ell} = \frac{2\sqrt{\frac{52}{5.2 \times 10^{-3}}}}{2}$$

$$f = 100 \text{ Hz}$$

$$11. y = 20 \sin 2\pi (100t) \cos 2\pi (0.02x)$$

$$\therefore A = 20, \text{ loop length} = \lambda/2 = \frac{50}{2} = 25 \text{ units}$$

$$13. L = (2n - 1) \frac{\lambda}{4}$$

$$\lambda = \frac{4L}{(2n-1)} \quad \lambda_1 = \frac{4L}{2(1)-1} = 4L$$

$$\lambda_2 = \frac{4L}{4-1} = \frac{4L}{3} \quad \lambda_3 = \frac{4L}{6-1} = \frac{4L}{5}$$

14. Since there is no relative motion between the listener and source, hence actual frequency will be heard by listener.

15. The linear velocity of whistle

$$v_s = r\omega = 1.2 \times 2\pi \frac{400}{60} = 50 \text{ m/s}$$

When whistle approaches the listener, heard frequency will be maximum and when whistle recedes away, heard frequency will be minimum

$$\text{So, } n_{\max} = n \left(\frac{v}{v - v_s} \right) = 500 \left(\frac{340}{290} \right) = 586 \text{ Hz}$$

$$n_{\min} = n \left(\frac{v}{v + v_s} \right) = 500 \left(\frac{340}{390} \right) = 436 \text{ Hz}$$

16. No change in frequency

17. At point A, source is moving away from observer so apparent frequency $n_1 < n$ (actual frequency) At point B source is coming towards observer so apparent frequency $n_2 > n$ and point C source is moving perpendicular to observer so $n_3 = n$

$$\text{Hence } n_2 > n_3 > n_1$$

18. When observer moves towards stationary source then apparent frequency

$$n' = \left[\frac{v + v_0}{v} \right] n = \left[\frac{v + v/5}{v} \right] n = \frac{6}{5} n = 1.2n$$

Increment in frequency = 0.2 n so percentage

$$\text{change in frequency} = \frac{0.2n}{n} \times 100 = 20\%$$

$$19. \text{ Velocity of wave, } v = \sqrt{\frac{\text{stress}}{\text{density}}} = \sqrt{\frac{3.2 \times 10^8}{8 \times 10^3}}$$

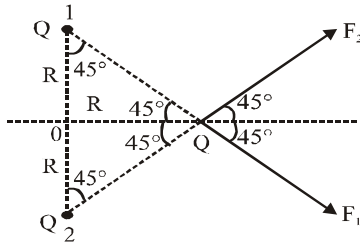
$$= \sqrt{0.4 \times 10^5} = \sqrt{4 \times 10^4} = 200 \text{ m/s}$$

$$\therefore \text{ fundamental frequency} = \frac{v}{2L} = \frac{200}{2 \times 1.25} = 80 \text{ Hz}$$

Electric Charge & Coulomb's Law : RACE # 01

SOLUTION

1.

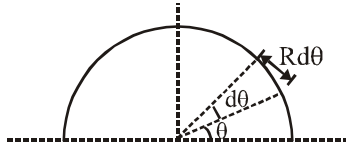


$$F_{\text{Net}} = \sqrt{F_1^2 + F_2^2}$$

$$F_1 = F_2 = \frac{kQ \cdot Q}{(R\sqrt{2})^2} = \frac{kQ^2}{2R^2}$$

$$F_{\text{Net}} = \sqrt{2} \frac{kQ^2}{2R^2} = \frac{\sqrt{2}Q^2}{8\pi\epsilon_0 R^2}$$

2.



Let us assume an element at angle ' θ ' & angular width ' $d\theta$ '

Q = Linear charge density $\times$ length

so charge on element

$$dQ = \lambda R d\theta = \lambda_0 \cos \theta R d\theta$$

$$\int_0^Q dQ = \lambda_0 R \int_0^\pi \cos \theta d\theta \Rightarrow Q = \lambda_0 R [\sin \theta]_0^\pi$$

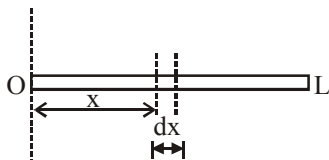
$$\Rightarrow Q = \lambda_0 R [\sin \pi - \sin 0] \Rightarrow [Q = 0]$$

Hence option (2)

3.

Let us assume an element of length dx at a distance x from O .

charge on element



$$dQ = \lambda dx = \frac{\lambda_0 x}{L} dx$$

$$\int_0^Q dQ = \int_0^L \frac{\lambda_0}{L} x dx \Rightarrow Q = \frac{\lambda_0}{L} \frac{L^2}{2}$$

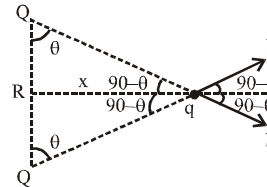
$$\Rightarrow Q = \frac{\lambda_0 L}{2}$$

Hence option (2)

$$F_{\text{elec}} = \frac{kq_1 q_2}{r^2}$$

$$F_{\text{grav}} = \frac{Gm_1 m_2}{r^2}$$

5.



$$F_{\text{Net}} = 2F \cos (90^\circ - \theta) = 2F \sin \theta$$

$$= \frac{2kQq}{\left\{\left(\frac{R}{2}\right)^2 + x^2\right\} \left\{\left(\frac{R}{2}\right)^2 + x^2\right\}^{1/2}}$$

$$\text{For } F \text{ to be max. } \frac{dF}{dx} = 0$$

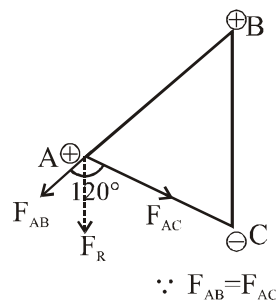
$$\Rightarrow \frac{dF}{dx} = 0 \Rightarrow \left\{\left(\frac{R}{2}\right)^2 + x^2\right\}^{-3/2} \frac{d}{dx} x + x \frac{d}{dx} \left\{\left(\frac{R}{2}\right)^2 + x^2\right\}^{-3/2} = 0$$

$$\Rightarrow \left(\frac{R}{2}\right)^2 + x^2 - x \frac{3}{2} (2x) = 0 \Rightarrow x = \frac{R}{2\sqrt{2}}$$

6.

As same electrostatic force and same gravitation force acts on each spheres, both will make equal angle with vertical under equilibrium.

7.



$$\therefore F_{AB} = F_{AC}$$

$$\angle = 120^\circ$$

so, F_R is in the $(-Y)$ -direction

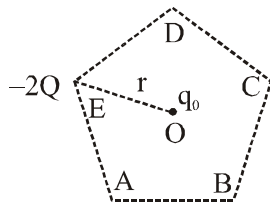
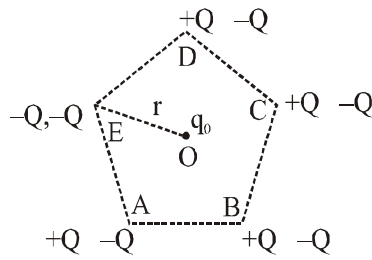
$$10. \frac{\epsilon}{\epsilon_0} = K$$

$$\epsilon = K \cdot \epsilon_0$$

$$\epsilon = 81 \times 8.85 \times 10^{-12}$$

$$\epsilon = 7.17 \times 10^{-10}$$

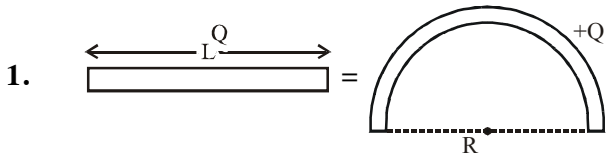
11.



Since, it is a regular polygon If all charges were positive net force would be zero. Now one positive is missing and one additional $-Q$ is placed at point E.

$$\therefore F_{\text{net}} = \frac{2kQq_0}{r^2}$$

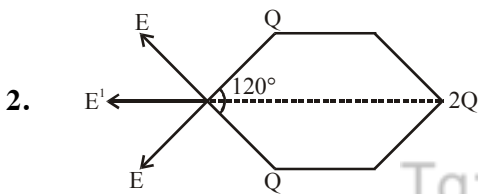
Tg: @Chalnaayaaar



$$E = \frac{2K\lambda}{R} \sin \frac{\theta}{2}$$

$$E = \frac{2K\lambda}{R} = \frac{2 \times 1}{4\pi\epsilon_0} \frac{Q}{L \times \frac{L}{\pi}} \quad [\pi R = L]$$

$$E = \frac{Q}{2\epsilon_0 L^2}$$



$$E_{\text{net}} = 2E \cos 60^\circ + E' = E + E'$$

$$= \frac{KQ}{a^2} + \frac{K(2Q)}{(2a)^2} = \frac{3}{2} \frac{KQ}{a^2}$$

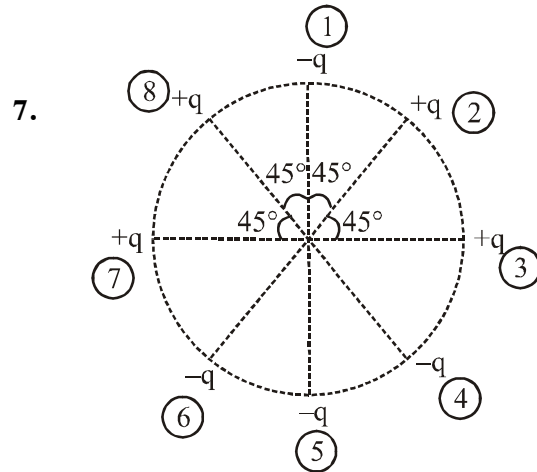
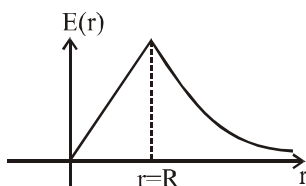
3. $\vec{E}_{\text{inside}} = \vec{0}$

5. For uniformly charged non conducting sphere

$$E = \frac{kQr}{R^2}, \text{ for } r \leq R$$

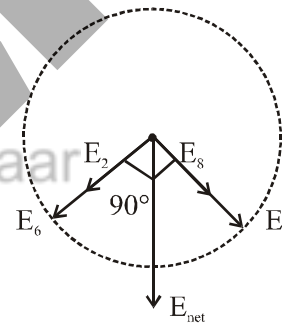
$$\Rightarrow E \propto r$$

$$E = \frac{kQ}{r^2}, \text{ for } r > R \Rightarrow E \propto \frac{1}{r^2}$$



Field at centre due to charge at point 1,5 & 3,7 will cancel each other

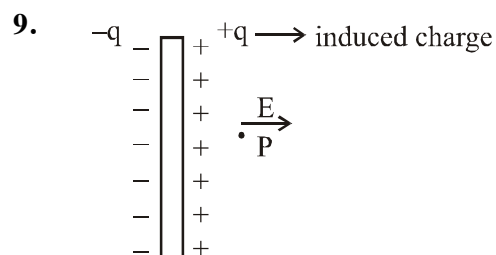
field due to charge at point (2), (6), (8) & (4) will be given as



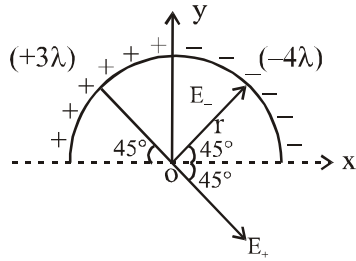
$$E_2 = E_6 = E_8 = E_4 = \frac{kq}{r^2}$$

$$E_{\text{net}} = \sqrt{2} \times \frac{2kq}{r^2} \quad E_{\text{net}} = \frac{q}{\sqrt{2}\pi\epsilon_0 r^2}$$

8. Electric field can deflect charged particles only



10.



$$E_+ = \frac{2k(3\lambda)}{r} \sin \frac{90}{2}$$

$$\Rightarrow \frac{3\sqrt{2} k\lambda}{r}$$

$$E_- = \frac{2k(4\lambda)}{r} \sin \frac{90}{2}$$

$$\Rightarrow \frac{4\sqrt{2} k\lambda}{r}$$

$$E_{\text{net}} = \sqrt{(E_+)^2 + (E_-)^2 + 2(E_+)(E_-)\cos 90}$$

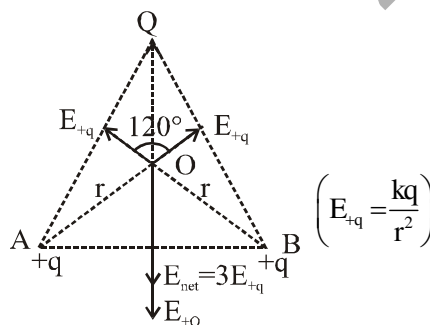
$$E_{\text{net}} = \sqrt{\left(\frac{3\sqrt{2}k\lambda}{r}\right)^2 + \left(\frac{4\sqrt{2}k\lambda}{r}\right)^2}$$

$$E_{\text{net}} = \frac{\sqrt{2} k\lambda}{r} \sqrt{9+16}$$

$$E_{\text{net}} = \frac{5\sqrt{2} k\lambda}{r} \Rightarrow \frac{5\sqrt{2}\lambda}{4\pi\epsilon_0 r} \text{ above } +x \text{ direction.}$$

(as $E_- > E_+$)

11. If Q is +ve

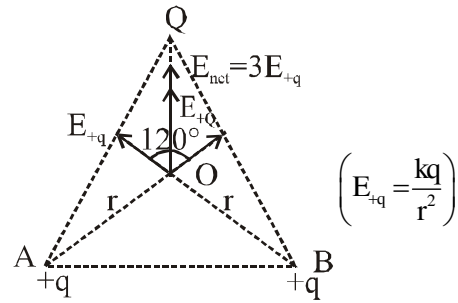


$$|\vec{E}_{+Q} + \vec{E}_{+q} + \vec{E}_{+q}| = 3E_{+q}$$

$$\Rightarrow \frac{kQ}{r^2} - \frac{kq}{r^2} = \frac{3kq}{r^2}$$

$$\Rightarrow Q = +4q$$

If Q is -ve



$$|\vec{E}_{+Q} + \vec{E}_{+q} + \vec{E}_{+q}| = 3E_{+q}$$

$$\Rightarrow \frac{kQ}{r^2} + \frac{kq}{r^2} = \frac{3kq}{r^2}$$

$$\Rightarrow Q = -2q$$

12. Distance of point A from origin

$$r = \sqrt{3^2 + 4^2} = 5$$

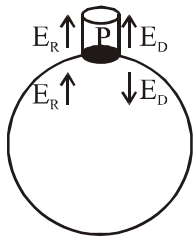
$$E = \frac{kQ}{r^2} = \frac{kQ}{25} \quad \dots(1)$$

Distance of point B from origin

$$r' = \sqrt{24^2 + 7^2} = 25$$

$$E' = \frac{kQ}{(r')^2} = \frac{kQ}{(25)^2} = \frac{kQ}{625} \Rightarrow \boxed{E' = \frac{E}{25}} \text{ from (1)}$$

11.



E_R = Due to shell

E_D = due to disc

$$E_{\text{out}} = \frac{\sigma}{\epsilon_0}$$

$$E_{\text{in}} = 0$$

$$E_R + E_D = \frac{\sigma}{\epsilon_0}$$

$$E_R - E_D = 0$$

$$E_R = E_D$$

$$2E_R = \frac{\sigma}{\epsilon_0}$$

$$E_R = \frac{\sigma}{2\epsilon_0} \text{ directed outwards.}$$

Tg: @Chalnaayaaar

1. by Gauss' law,

$$E(4\pi r_B^2) = \frac{Q_2}{\epsilon_0}$$

$$V = V_{R_1} + V_{R_2}$$

$$= \frac{kQ_1}{R_1} + \frac{kQ_2}{r_B}$$

$$2. \quad V_{\text{centre}} = 8 \left(\frac{kq}{\left(\frac{a\sqrt{3}}{2} \right)} \right) = \frac{16q}{4\pi\epsilon_0 a\sqrt{3}} = \frac{4q}{\sqrt{3}\pi\epsilon_0 a}$$

$$W = -q [v_{\infty} - v_{\text{centre}}] = \frac{4q^2}{\sqrt{3}\pi\epsilon_0 a}$$

$$3. \quad \frac{kQ}{2^2} = 4 \Rightarrow kQ = 16$$

$$\text{so, } V = \frac{kQ}{2} = 8$$

$$W = qV = 8J$$

$$4. \quad U = \frac{k(1)(-1)}{2} + \frac{k(1)(-1)}{2} + \frac{k(1)(1)}{1} = 0$$

$$5. \quad V = \frac{kQ}{R} + \frac{k(3Q)}{R} + \frac{k(-2Q)}{R}$$

$$= \frac{2kQ}{R} = \frac{Q}{2\pi\epsilon_0 R}$$

6. For hollow metal sphere, $V_{\text{centre}} = V_{\text{surface}}$

$$7. \quad W = U_f - U_i = \frac{Kq^2}{a}$$

$$8. \quad U = \frac{2kQq}{a} + \frac{kq^2}{\sqrt{2}a} = 0$$

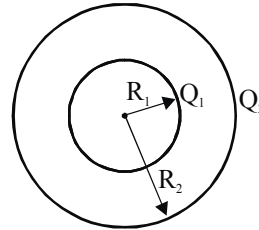
$$9. \quad W = U_f - U_i = 0 - 0 = 0$$

$$10. \quad V_{\text{centre}} = \frac{3}{2} \frac{kQ}{R}$$

$$V_r = \frac{1}{2} \left(\frac{3}{2} \frac{kQ}{R} \right) = \frac{kQ}{r} \Rightarrow r = \frac{4}{3} R$$

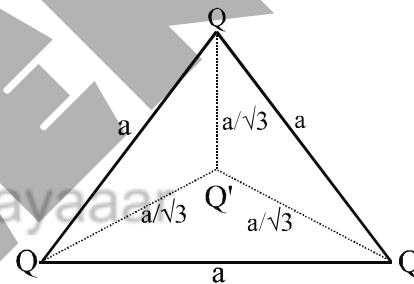
so, distance from surface = $R/3$

11.



$$P.D. = kQ_1 \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \Rightarrow P.D. \propto Q_1$$

12.



Self energy

$$= 3 \left[\frac{kQ^2}{a} + \sqrt{3}k \frac{QQ'}{a} \right] = 0$$

$$\Rightarrow Q = -\sqrt{3}Q' \Rightarrow Q' = -\frac{Q}{\sqrt{3}}$$

13. $\therefore W = q \Delta V \Rightarrow W = 0$ (as $V_i = V_f$)

$$14. \quad W = q\Delta V$$

$$= q(V_{\infty} - V_C)$$

$$= q \left[0 - 3 \frac{kq}{(a/\sqrt{3})} \right] = \frac{-3\sqrt{3}kq^2}{a}$$

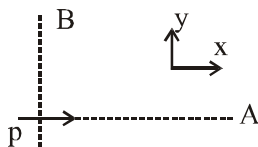
ELECTRIC DIPOLE : RACE # 05

SOLUTION

1. Torque $\vec{\tau} = \vec{p} \times \vec{E}$ then torque on $\vec{p}_2$ due to $\vec{p}_1 = \vec{p}_2 \times \vec{E}_1$ but angle between $\vec{p}_2$ and $\vec{E}_1$ is 180° so $\vec{\tau}_{21} = \vec{0}$

2. $F = -p_2 \left(\frac{dE}{dr} \right) = -\frac{6kp_1p_2}{r^4}$
 $p_1 \cdot p_2 > 0 \Rightarrow F < 0$ attraction
 $p_1 \cdot p_2 < 0 \Rightarrow F > 0$ repulsion

3. $F = -\frac{6kp_1p_2}{r^4} \quad [F \propto r^{-4}]$

4. 

$$\vec{E}_A = \frac{2kp}{r^3}(\hat{i})$$

$$\vec{E}_B = \frac{kp}{r^3}(-\hat{i}) \Rightarrow \vec{E}_A = -2\vec{E}_B$$

5. $\vec{p}_x = (-2q)(-a)\hat{i} + (-2q)(a)\hat{i}$
 $+ (3q)(0)\hat{i} + (q)(0)\hat{i}$

$$\vec{p}_y = (-2q)(0)\hat{i} + (-2q)(0)\hat{j}$$

$$+ (3q)(a)\hat{j} + (q)(-a)\hat{j}$$

6. net force on dipole in uniform electric field is always zero.

7. E_{ext} at $-q > E_{\text{ext}}$ at $(+q)$

8. Max. P.E = $+pE = U$
W.D = $pE [\cos 180^\circ - \cos 0^\circ]$
 $= -2pE = -2U$

9. $V = \frac{kp \cos \theta}{r^2} \Rightarrow V \propto \cos \theta$

10. $\vec{E}_0 = \frac{2kp}{r^3}(\hat{p}) \quad \dots(1)$

$$\vec{E} = \frac{kp}{(2r)^3}(-\hat{p})$$

$$\vec{E} = -\frac{1}{8} \left(\frac{kp}{r^3} \right) \hat{p}$$

$$\vec{E} = -\frac{\vec{E}_0}{16} \text{ from (1)}$$

MISCELLANEOUS : RACE # 06

SOLUTION

1. At closest approach

$$\frac{1}{2}mv^2 = \frac{(2e)(Ze)}{4\pi\epsilon_0 d_{\min}} \Rightarrow d_{\min} \propto \frac{Ze^2}{mv^2}$$

2. $E = \frac{F}{q} = \frac{3000}{3} = 1000 \text{ V/m}$

$$\Delta V = Ed = 1000 \times \frac{1}{100} = 10 \text{ V}$$

3. E_x is slope of $V-x$ graph.

4. $V_A - V_B = -\vec{E} \cdot (\vec{r}_A - \vec{r}_B)$

$$= - (2\hat{i} + 3\hat{j}) \cdot (\hat{i} + 2\hat{j} - (2\hat{i} + \hat{j} + 3\hat{k}))$$

$$= 2 - 3$$

$$= -1 \text{ V}$$

5. $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

At (1, 1, 1)

$$\vec{E} = -4\hat{k}\hat{i} + 2\hat{k}\hat{j} - 2\hat{k}\hat{k}$$

$$E = \sqrt{(4k)^2 + (2k)^2 + (2k)^2} = 2k\sqrt{6}$$

6. Direction of electric field is from higher potential to lower potential.

7. $E \propto r \Rightarrow E = kr$

$$\int_0^v dV = - \int_0^r E dr \Rightarrow v = - \int_0^r kr dr \Rightarrow v = - \frac{kr^2}{2}$$

$$v \propto r^2$$

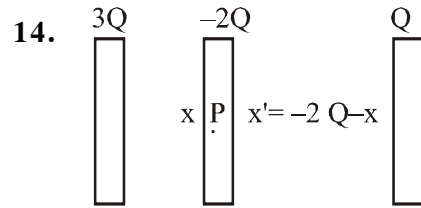
8. Entire charge of inner sphere will flow to outer shell so that their potential is same.

9. Distribution will be such to give, $\vec{E}_{\text{net}}$ inside = 0

10. $\sigma \propto \frac{1}{r}$ for conductors

11. Total flux = $\frac{q_{\text{net}}}{\epsilon_0} = \frac{0}{\epsilon_0}$ ($q_{\text{net}} = 0$ for dipole)

12. $E = 0$ outside as $q_{\text{enc}} = 0$ (by gauss theorem)



for E_P to be zero $\frac{3Q + x}{2\epsilon_0 A} = \frac{-2Q - x + Q}{2\epsilon_0 A}$

$$3Q + x = -Q - x$$

$$x = -2Q$$

$$x' = 0$$

15. by volume conservation

$$R^3 = 64 r^3$$

$$\Rightarrow R = 4r$$

$$\frac{\sigma_r}{\sigma_R} = \frac{q}{Q} \times \frac{R^2}{r^2}$$

$$= \frac{1}{64} \times 16 = \frac{1}{4}$$

by charge conservation

$$Q = 64q \quad \sigma_r : \sigma_R :: 1 : 4$$

- 16.

by induction

Total charge on outer surface = $Q + q$

17. electrostatic shielding

18. $\tan \theta = \frac{Fe}{mg} \Rightarrow \theta \propto \frac{1}{m}$

this if $\alpha > \beta \Rightarrow m_1 < m_2$

19. $T = 2\pi\sqrt{\frac{\ell}{g}} ; T^1 = 2\pi\sqrt{\frac{\ell}{g - \frac{qE}{m}}} \Rightarrow T^1 > T$

$$1. \quad v_d = \frac{eE\tau}{m}$$

$$v_d = \frac{e(V)\tau}{m\ell} \quad \left[\because E = \frac{V}{\ell} \right]$$

$$\therefore v_d \propto V$$

$$2. \quad R = \rho \frac{\ell}{A} = \frac{\rho \ell^2}{V} = \frac{\rho \ell^2}{m/\sigma}$$

where $\sigma \rightarrow$ density of copper

$$R = \rho \sigma \frac{\ell^2}{m} \text{ So } R_1 : R_2 : R_3 = \frac{\ell_1^2}{m_1} : \frac{\ell_2^2}{m_2} : \frac{\ell_3^2}{m_3}$$

$$= \frac{5^2}{1} : \frac{3^2}{3} : \frac{1^2}{5}$$

$$R_1 : R_2 : R_3 = 125 : 15 : 1$$

Hence option (3)

3. For small changes

$$\begin{aligned} \Delta R\% &= 4(\Delta d\%) \\ &= 4(0.01\%) \\ &= 0.04\% \end{aligned}$$

4. Let R_0 = Resistance of wire at 0°C

$$\Rightarrow 6 = R_0 [1 + \alpha (100-0)] \quad \text{..(1)}$$

$$5 = R_0 [1 + \alpha (50-0)] \quad \text{..(2)}$$

$$\text{from (1), (2)} \Rightarrow \frac{6}{5} = \frac{1+100\alpha}{1+50\alpha} \Rightarrow \alpha = \frac{1}{200} ^\circ\text{C}^{-1}$$

$$\begin{aligned} \text{put in (1)} \quad 6 &= R_0 \left[1 + \frac{1}{200} \times 100 \right] \\ &\Rightarrow R_0 = 4\Omega \end{aligned}$$

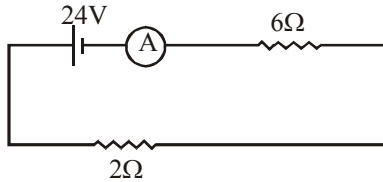
Current Electricity : Combination of Resistance, KVL & KCL : RACE # 02

SOLUTION

1. The effective resistance of two parallel resistor is closer to the smaller value.

Hence option (1)

2. The circuit can be redrawn as



$$I = \frac{24}{6+2} = 3A$$

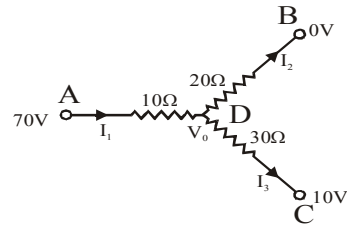
3. Let the potential at junction be x
 $\therefore$ by nodal Analysis

$$\frac{x-20}{2} + \frac{x-5}{4} + \frac{x}{2} = 0$$

$$\therefore x = 9V$$

$$I = \frac{x}{2} = \frac{9}{2} = 4.5A$$

4. $I_1 = I_2 + I_3$



$$\Rightarrow \frac{70 - V_0}{10} = \frac{V_0 - 10}{30} + \frac{V_0 - 0}{20}$$

$$\Rightarrow 420 - 6V_0 = 2V_0 - 20 + 3V_0 \Rightarrow V_0 = 40V$$

$$I_{AD} = \frac{70 - 40}{10} A = 3A, I_{DB} = 2A \text{ \& } I_{DC} = 1A$$

5. $V_C + 5 + 12 - 12 - 3 - 8 = V_B$

$$V_C - V_B = 6V$$

6. Potential difference between A and B

$$V_A - V_B = 1 \times 1.5$$

$$\Rightarrow V_A - 0 = 1.5V \Rightarrow V_A = 1.5V$$

Potential difference between B and C

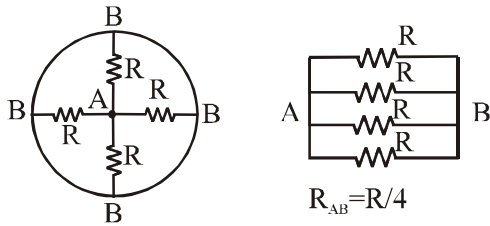
$$V_B - V_C = 1 \times 2.5 = 2.5V$$

$$\Rightarrow 0 - V_C = 2.5V \Rightarrow V_C = -2.5V$$

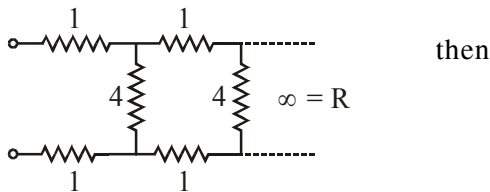
Potential difference between C and D

$$V_C - V_D = -2V \Rightarrow -2.5 - V_D = -2 \Rightarrow V_D = -0.5V.$$

1.



2.



$$R = \frac{4R}{4+R} + 2 \Rightarrow R = \frac{4R}{4+R} + 2$$

$$\Rightarrow 4R + R^2 = 4R + 2(4 + R)$$

$$\Rightarrow R^2 + 4R - 6R - 8 = 0$$

$$\Rightarrow R^2 - 4R + 2R - 8 = 0$$

$$\Rightarrow R = 4 \text{ or } R = -2 \text{ Hence option (1)}$$

3.

Apply KVL from A to B

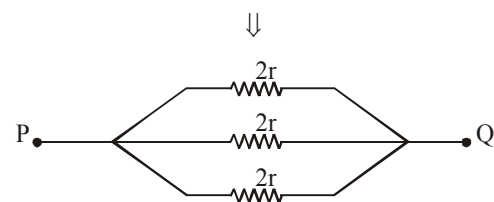
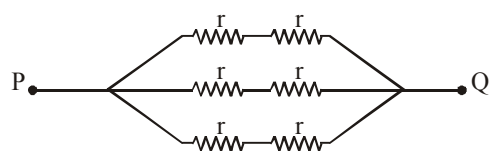
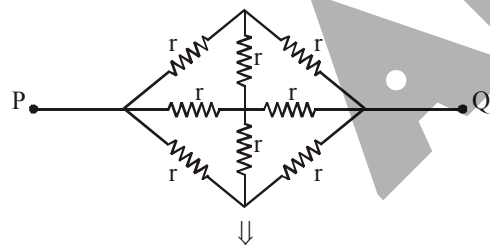
$$-25 + 2 \times 2 + 2 \times R + 2 \times 8 = 0$$

$$R = 2.5$$

Hence option (4)

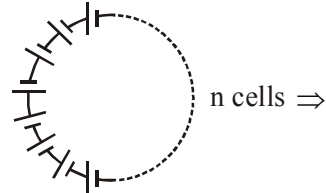
4.

The given circuit can be simplified as follows

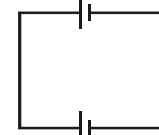


$$R_{PQ} = \frac{2r}{3} = \frac{2}{3} \times \frac{3}{2} = 1\Omega$$

5.



$$(n-2)\varepsilon, (n-2)r$$



$$2\varepsilon, 2r$$

$$I = \frac{(n-2)\varepsilon - 2\varepsilon}{nr} = \left(1 - \frac{4}{n}\right) \frac{\varepsilon}{r}$$

Potential across any one reversed Polarity cell

$$V = \varepsilon + Ir$$

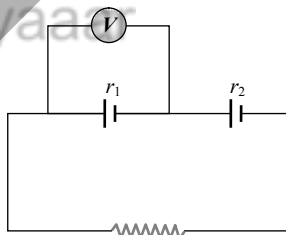
$$= \varepsilon + \left(1 - \frac{4}{n}\right) \varepsilon = 2\varepsilon \left(1 - \frac{2}{n}\right)$$

Hence option (4)

6.

Let the voltage across any one cell is V, then

$$V = E - ir = E - r_1 \left(\frac{2E}{r_1 + r_2 + R} \right)$$



But $V = 0$

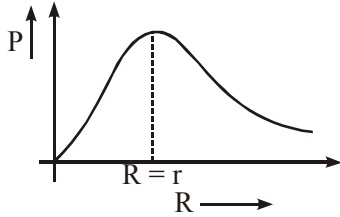
$$\Rightarrow E - \frac{2Er_1}{r_1 + r_2 + R} = 0$$

$$\Rightarrow r_1 + r_2 + R = 2r_1$$

$$\Rightarrow R = r_1 - r_2$$

Current Electricity : Power & Heating Effect : RACE # 04 SOLUTIONS

1. Variation of power with load resistance is



hence power consumed will decrease.

Hence option (1) is correct.

2. In series $20 = \frac{P}{n}$; In parallel $P_n = 2000$

$$\Rightarrow P = 200 \text{ \& } n = 10$$

Hence option (4) is correct

3. For parallel coils

$$\frac{1}{t_{\text{eff}}} = \frac{1}{t_1} + \frac{1}{t_2} \Rightarrow t_{\text{eff}} = \frac{10 \times 15}{10 + 15} = 6 \text{ mins.}$$

Hence option (2) is correct.

4. House hold connections are parallel combinations so :-

$$P_1 > P_2 \Rightarrow \frac{V^2}{R_1} > \frac{V^2}{R_2} \Rightarrow R_1 < R_2$$

So dim bulb will have larger resistance

Hence option (2) is correct.

5. In series current will be same so

$$\frac{P_1}{P_2} = \frac{I^2 R_1}{I^2 R_2} = \frac{R_1}{R_2} = \frac{1}{2}$$

Hence option (1) is correct.

$$\frac{R_1}{R_2} = \frac{P_2}{P_1} = \frac{5}{3}$$

- 6.

$$\begin{aligned} \text{So voltage drop across 60 W bulb} &= 220 \times \frac{5}{8} \\ &= 137.5 \text{ V} \\ &> 110 \text{ V} \end{aligned}$$

So 60 W bulb will fuse

Hence option (1) is correct.

Current Electricity : Electrical Instruments : RACE # 05

SOLUTIONS

1. Applying KVL

$$12 - 2I - 4I - 6 = 0 \Rightarrow I = 1A$$

$$\text{So voltmeter reading} = 12 - 2I = 10 \text{ V.}$$

Hence option (1)

2. Before shunting

$$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3} \quad \dots(1)$$

After shunting

$$\frac{R_1}{R'_2} = \frac{50}{50} = \frac{1}{1} \quad \dots(2)$$

from eq. (1) & (2)

$$\frac{R'_2}{R_2} = \frac{2}{3}$$

$$\text{Now } R'_2 = \frac{R_2(10)}{R_2 + 10}$$

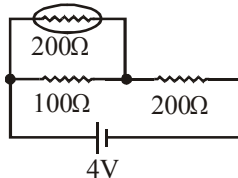
$$\text{so } \frac{10}{10 + R_2} = \frac{2}{3} \Rightarrow R_2 = 5\Omega$$

From eq. (i)

$$R_1 = R_2 \left(\frac{2}{3} \right) = \frac{10}{3} \Omega$$

Hence option (1)

3. voltmeter



Effective resistance of circuit

$$R_{\text{eff}} = 200 + \frac{200 \times 100}{200 + 100} = \frac{800}{3} \Omega$$

$$\text{current in circuit } I_{\text{eff}} = \frac{4}{\frac{800}{3}} = \frac{3}{200} \text{ A}$$

$$\begin{aligned} \text{so voltage across the voltmeter} &= 4 - \left(\frac{3}{200} \times 200 \right) \\ &= 1 \text{ volt} \end{aligned}$$

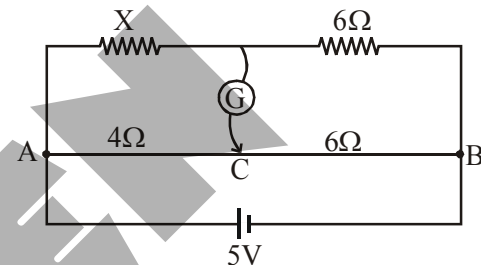
Hence option (1)

4. From potentiometer principle

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{60}{40} = \frac{3}{2}$$

$$\text{so } \frac{E_1}{E_2} = \frac{3+2}{3-2} = \frac{5}{1} \quad \text{Hence option (2)}$$

5..



Resistance of the part AC

$$R_{AC} = 0.1 \times 40 = 4\Omega \quad \text{and} \quad R_{CB} = 0.1 \times 60 = 6\Omega$$

$$\text{In balanced condition } \frac{X}{6} = \frac{4}{6} \Rightarrow X = 4\Omega$$

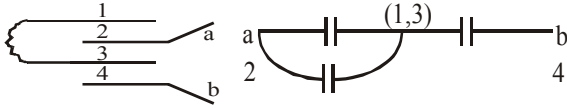
Equivalent resistance $R_{\text{eq}} = 5\Omega$ so current

$$\text{drawn from battery } i = \frac{5}{5} = 1A.$$

CAPACITOR (PARALLEL PLATE CAPACITOR) : RACE # 01

SOLUTIONS

- Charge on the plates will be same in magnitude but different in nature.
- circuit can be redrawn as



where $C = \frac{\epsilon_0 A}{d}$ & $C_{\text{eff}} = \frac{2C}{3} = \frac{2}{3} \frac{\epsilon_0 A}{d}$

- Series connection

$$Q = C_{\text{eq}} V = \frac{20C}{3}$$

$$C = 6 \mu\text{F}$$

$$Q = 40 \mu\text{C}$$

- $C = \frac{\epsilon_0 A}{d}$
A is common area

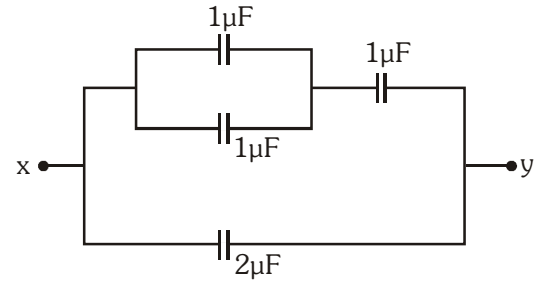
So $C = \frac{\epsilon_0 A_2}{d}$

$$5. \quad C_{\text{eq}} = \frac{2 \times 1}{2 + 1} = \frac{2}{3} \mu\text{F} \text{ (series)}$$

$$V = 120 \text{ V}$$

$$\Rightarrow Q = CV = 80 \mu\text{C}$$

-



$$C_{\text{eq}} = \frac{8}{3} \mu\text{F}$$

$$7. \quad Q_{4\mu\text{F}} = \left(\frac{4 \times 6}{4 + 6} \right) (10) \mu\text{C}$$

$$Q_{4\mu\text{F}} = 24 \mu\text{C}$$

Tg: @Chalnaayaaar

CAPACITOR (DIELECTRIC/SPHERICAL/RC CIRCUIT) : RACE # 02 SOLUTIONS

1. Since both capacitor are identical, common potential = $\frac{V}{2}$ and charge on both capacitors

$$Q_1 = Q_2 = \frac{1}{2}CV$$

$$\text{Energy stored by both capacitors } E_1 = E_2 = \frac{1}{8}CV^2 = \frac{E}{4}$$

2. Capacity of system increases so charge on A increase. Therefore, the energy of capacitor A will increase

$$3. \quad C = \frac{K_1 \epsilon_0 \left(\frac{A}{2}\right)}{d} + \frac{K_2 \epsilon_0 \left(\frac{A}{2}\right)}{d}$$

$$C = 30 \mu\text{F}$$

4. $V = Ed$

$$V = \text{constant}$$

$$E = \text{constant}$$

5. Current in 2Ω is 1 amp.
Potential difference across capacitor is 2 volt
 $Q = CV = 10 \mu\text{C}$

$$6. \quad \tau = R_{eq}C = \left(\frac{6R}{5}\right)C = \frac{6RC}{5}$$

$$7. \quad q = Q_0(1 - e^{-t/\tau}) = \frac{Q_0}{10}$$

$$\Rightarrow \frac{1}{10} = 1 - e^{-t/\tau}$$

$$\Rightarrow -\frac{t}{\tau} = \ln\left(\frac{9}{10}\right)$$

$$\Rightarrow t = \tau \ln\left(\frac{10}{9}\right)$$

Tg: @Chinnaayaaar

CAPACITOR (MISCELLANEOUS) : RACE # 03

SOLUTIONS

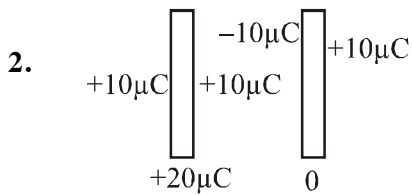
1. $C = \frac{A\epsilon_0}{d}$ [Parallel-plate capacitor]

$A = \frac{Cd}{\epsilon_0} \quad \because v = Ed \text{ \& } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$

$A = \frac{CV}{E\epsilon_0} = \frac{Q}{E\epsilon_0}$

$A = \frac{4\pi \times 9 \times 10^9 \times 12 \times 10^{-6}}{3 \times 10^6}$

$A = 0.45 \text{ m}^2$



$C = \frac{Q}{V}$

$\therefore V = \frac{Q}{C} = \frac{10\mu\text{C}}{10\mu\text{F}} = 1\text{V}$

3. $E = \frac{E_0}{K}$

$E_1 = \frac{E_0}{2} \text{ and } E_2 = \frac{E_0}{4}$

4. $\text{Power} = \frac{\text{Energy}}{\text{Time}}$

$P = \frac{\frac{1}{2}CV^2}{t}$

$P = 10 \text{ KW}$

Magnetic effect of current and Magnetism : RACE # 01

SOLUTION

$$1. \quad B = \frac{\mu_0 I}{2R} \left(\frac{\theta}{2\pi} \right) \left[1 - \frac{1}{2} + \frac{1}{3} \right] = \frac{5}{6} \frac{\mu_0 I}{4\pi R} \left(\frac{\theta}{R} \right)$$

$$2. \quad B \text{ due to straight wire portion} = 2 \times \frac{\mu_0 I}{4\pi R} (-\hat{j})$$

$$B \text{ due to semicircle} = \frac{\mu_0 I}{4R} (-\hat{k})$$

$$B_{\text{Net}} = \frac{\mu_0 I}{4R} \sqrt{\frac{4}{\pi^2} + 1} = \frac{\mu_0 I}{4\pi R} \sqrt{\pi^2 + 4}$$

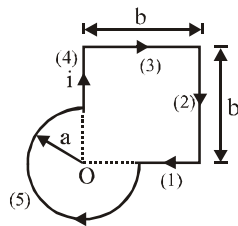
$$3. \quad B_1 = B_4 = 0$$

$$B_2 = B_3 = \frac{\mu_0 i}{4\pi b} \sin 45^\circ \otimes$$

$$\& B_5 = \frac{\mu_0 i}{2a} \left[\frac{3}{4} \right] \otimes$$

$$B_{\text{net}} = B_2 + B_3 + B_5$$

$$\text{so } B_{\text{Net}} = \frac{\mu_0 i}{2\sqrt{2}\pi b} + \frac{3\mu_0 i}{8a} \otimes$$



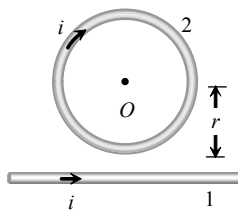
4. The given shape is equivalent to the following diagram

The field at O due to straight part of conductor

is $B_1 = \frac{\mu_0 I}{2\pi r} \otimes$. The field at O due to circular

coil is $B_2 = \frac{\mu_0 I}{2r} \otimes$.

$$B_2 > B_1$$



$$\text{i.e. } B = B_2 - B_1 = \frac{\mu_0 I}{2\pi r} (\pi - 1)$$

$$5. \quad B = \frac{\mu_0 I}{2\pi r} \Rightarrow \frac{B_1}{B_2} = \frac{r_2}{r_1} \Rightarrow \frac{10^{-8}}{B_2} = \frac{12}{4}$$

$$\Rightarrow B_2 = 3.33 \times 10^{-9} \text{ Tesla}$$

6. Two coils carrying current in opposite direction, hence net magnetic field at centre will be difference of the two fields.

i.e.

$$B_{\text{net}} = \frac{\mu_0 N}{2} \left[\frac{i_1}{r_1} - \frac{i_2}{r_2} \right] = \frac{10\mu_0}{2} \left[\frac{0.2}{0.2} - \frac{0.3}{0.4} \right] = \frac{5}{4} \mu_0$$

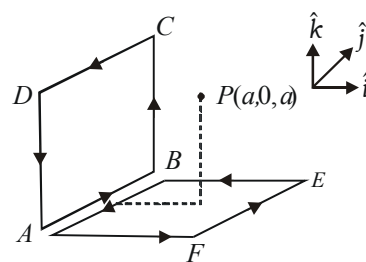
7. Magnetic field at the centre of a circuit loop will be zero, if wire is uniform.

8. The magnetic field at $P(a, 0, a)$ due to the loop is equal to the vector sum of the magnetic fields produced by loops ABCDA and AFEBA as shown in the figure.

Magnetic field due to loop ABCDA will be along $\hat{i}$ and due to loop AFEBA, along $\hat{k}$. Magnitude of magnetic field due to both the loops will be equal.

Therefore, direction of resultant magnetic field

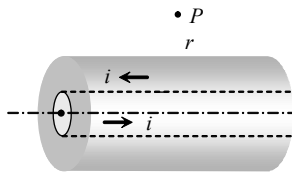
at P will be $\frac{1}{\sqrt{2}} (\hat{i} + \hat{k})$.



Magnetic effect of current and Magnetism : RACE # 02

SOLUTION

1. The respective figure is shown below Magnetic field at P due to inner and outer conductors are equal and opposite. Hence net magnetic field at P will be zero.



2. The magnetic field in the solenoid along its axis

(i) At an internal point = $\mu_0 ni$

$$= 4\pi \times 10^{-7} \times 5000 \times 4 = 25.1 \times 10^{-3} \text{ Wb/m}^2$$

(Here $n = 50 \text{ turns/cm} = 5000 \text{ turns/m}$)

(ii) At one end

$$B_{\text{end}} = \frac{1}{2} B_{\text{in}} = \frac{\mu_0 ni}{2} = \frac{25.1 \times 10^{-3}}{2} = 12.6 \times 10^{-3} \text{ Wb/m}^2$$

$$3. \quad B = \frac{\mu_0 \mu_r Ni}{2\pi r} \Rightarrow 1 = \frac{4\pi \times 10^{-7} \times \mu_r \times 400 \times 2}{0.4}$$

$$\Rightarrow \mu_r \approx 400 (\text{appx.})$$

$$4. \quad \text{By using } B = \frac{\mu_0 i}{2\pi r} \left(\frac{r^2 - a^2}{b^2 - a^2} \right)$$

$$\text{here } r = \frac{3R}{2}, a = R, b = 2R$$

$$\Rightarrow B = \frac{\mu_0 i}{2\pi \left(\frac{3R}{2} \right)} \times \left\{ \frac{\left(\frac{3R}{2} \right)^2 - R^2}{(2R)^2 - R^2} \right\} = \frac{5\mu_0 i}{36\pi R}$$

Magnetic effect of current and Magnetism : RACE # 03

SOLUTION

$$1. \quad r = \frac{\sqrt{2mK}}{qB} \text{ i.e. } r \propto \frac{\sqrt{m}}{q}$$

Here kinetic energy K and B are same.

$$\therefore \frac{r_p}{r_\alpha} = \frac{\sqrt{m_p}}{\sqrt{m_\alpha}} \cdot \frac{q_\alpha}{q_p} = \frac{\sqrt{m_p}}{\sqrt{4m_p}} \cdot \frac{2q_p}{q_p} = 1$$

$$2. \quad r \propto \frac{1}{B} \text{ i.e. } \frac{r_1}{r_2} = \frac{B_2}{B_1} \Rightarrow r_2 = \frac{B_1}{B_2} \times r = 2r$$

$$3. \quad \text{Time period of proton } T_p = \frac{2\pi m}{qB} = 5\mu \text{ sec}$$

By using

$$T = \frac{2\pi m}{qB} \Rightarrow \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha} = \frac{4m_p}{m_p} \times \frac{q_p}{2q_p} = 2$$

$$\Rightarrow T_\alpha = 2T_p = 10\mu \text{ sec.}$$

$$4. \quad F = qvB \sin \theta \Rightarrow B = \frac{F}{qv \sin \theta}$$

$$B_{\min} = \frac{F}{qv} \quad (\text{when } \theta = 90^\circ)$$

$$\therefore B_{\min} = \frac{F}{qv} = \frac{10^{-10}}{10^{-12} \times 10^5} = 10^{-3} \text{ Tesla in } \hat{z} \text{ - direction.}$$

$$5. \quad \text{Using } eE = evB$$

$$\Rightarrow E = vB = 5 \times 10^6 \times 0.02 = 10^5 \text{ Vm}^{-1}$$

$$6. \quad r = \frac{mv}{qB} \Rightarrow \frac{r_1}{r_2} = \frac{m_1 v_1}{m_2 v_2} \times \frac{q_2}{q_1} = \frac{1 \times 2}{1 \times 3} \times \frac{2}{1} = \frac{4}{3}$$

$$7. \quad F = Bil \sin \theta$$

$$= 500 \times 10^{-4} \times 3 \times (40 \times 10^{-2}) \times \frac{1}{2} = 3 \times 10^{-2} \text{ N}$$

8. For no force on wire C, force on wire C due to wire D = force on wire C due to wire B

$$\Rightarrow \frac{\mu_0}{4\pi} \times \frac{2 \times 15 \times 5}{x} \times l = \frac{\mu_0}{4\pi} \times \frac{2 \times 5 \times 10}{(15-x)} \times l \Rightarrow x = 9 \text{ cm.}$$

$$9. \quad \vec{F}_{CAD} = \vec{F}_{CD} = \vec{F}_{CED}$$

$$\therefore \text{Net force on frame} = 3\vec{F}_{CD}$$

$$= (3)(2)(1)(4) \quad (F = ilB)$$

$$= 24 \text{ N}$$

10. Magnetic force will be towards or away from the centre depending upon the nature of charge and direction of magnetic field [$\otimes B$ or $\odot B$]

$$11. \quad F \propto v \quad \& \quad v \propto \sqrt{V_{\text{acc}}}$$

$$\text{so } \frac{F_1}{F_2} = \frac{\sqrt{V}}{\sqrt{2V}} = \frac{1}{\sqrt{2}} \text{ so } F_2 = \sqrt{2} F_1 = \sqrt{2} F$$

$$12. \quad T^1 = \frac{T}{4} = \frac{1}{4} \left(\frac{2\pi m}{qB} \right)$$

$$T^1 = \frac{\pi}{2} \times \frac{1.65 \times 10^{-27}}{1.6 \times 10^{-19} \times 100 \times 10^{-3}} = 1.62 \times 10^{-7}$$

$$13. \quad \text{Here } \vec{F}_{\text{net}} = \vec{0} \Rightarrow q\vec{E} + q(\vec{v} \times \vec{B}) = \vec{0}$$

$$\Rightarrow \vec{E} = \vec{B} \times \vec{v}$$

Therefore $\vec{E} \perp \vec{B}$ and $\vec{E} \perp \vec{v}$.

$$14. \quad \text{Energy in cyclotron } E = \frac{q^2 B^2 r^2}{2m}$$

$$\text{so } E \propto \frac{q^2}{2m} \Rightarrow \frac{E_1}{E_2} = \left(\frac{q_1}{q_2} \right)^2 \left(\frac{m_2}{m_1} \right)$$

For proton & deuteron $q_1 = q_2 = e$, $m_1 = 1 \text{ amu}$, $m_2 = 2 \text{ amu}$

$$\text{so } \frac{E_1}{E_2} = \frac{2}{1} \Rightarrow E_1 = 2E_2 = 80 \text{ MeV}$$

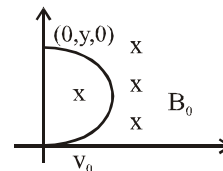
15. For the charge particle not to hit the wall

$$d = R$$

$$\Rightarrow d = \frac{mv}{qB} \Rightarrow B = \frac{mv}{qd} = \frac{v}{\frac{q}{m}d}$$

$$\text{so } B = \frac{v}{sd}$$

16. From the diagram



$$y = 2R$$

$$y = \frac{2mv}{qB_0}$$

$$\text{or } y = \frac{2v_0}{\alpha B_0}$$

17. If $d > R$ then the particle will deviate by 180°
After covering semicircle

18. given $v < \frac{qB\ell}{m}$

$$\frac{mv}{qB} < \ell \quad \text{so } R < \ell$$

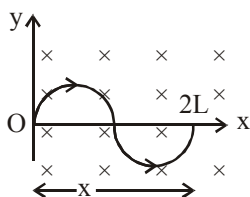
so The particle will not enter in region III

19. Tension generated in circular loop $T = IBR$

20. $|\vec{L}| = 2L$

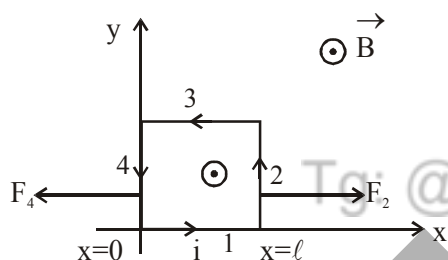
$$\text{so } F = iB |\vec{L}|$$

$$F = 2iBL$$



21. For wire 1 & 3

B is same & current is in opposite direction so forces will cancel each other



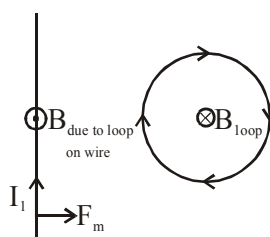
$$F_{\text{Net}} = F_2 - F_4 \quad \& \quad \vec{F}_1 + \vec{F}_3 = \vec{0}$$

$$F_{\text{Net}} = i B_0 \left(1 + \frac{\ell}{\ell}\right) \ell - i B_0 \left(1 + \frac{0}{\ell}\right) \ell$$

$$F_{\text{Net}} = 2iB_0\ell - iB_0\ell$$

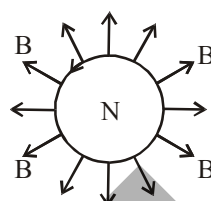
$$F_{\text{Net}} = iB_0\ell \quad \text{in } +x \text{ direction}$$

23.



25. Force will be into the plane &

$$F = iB \oint d\ell = iB (2\pi a) \otimes$$



Magnetic effect of current and Magnetism : RACE # 04

SOLUTION

1. Field at a point x distance from the centre of a current carrying loop on the axis is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3} = \frac{10^{-7} \times 2 \times 2.1 \times 10^{-25}}{(10^{-10})^3}$$

$$= 4.2 \times 10^{-32} \times 10^{30} = 4.2 \times 10^{-2} \text{ W/m}^2$$

2. $B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} \Rightarrow 12.56 = 10^{-7} \times \frac{2\pi \times i}{5.2 \times 10^{-11}}$

$$\Rightarrow i = 1.04 \times 10^{-3} \text{ A}$$

3. $\frac{\mu_0 I}{2\pi r} = B_H$

$$\therefore i = \frac{7 \times 0.05 \times 10^{-5}}{2 \times 3.142 \times 10^{-7}} = \frac{35}{2 \times 3.142} = 5.6 \text{ amp}$$

4. Suppose length of each wire is l .

$$A_{\text{square}} = \left(\frac{l}{4}\right)^2 = \frac{l^2}{16}$$

$$A_{\text{circle}} = \pi r^2 = \pi \left(\frac{l}{2\pi}\right)^2 = \frac{l^2}{4\pi}$$

$\therefore$ Magnetic moment

$$M = iA$$

$$\Rightarrow \frac{M_{\text{square}}}{M_{\text{circle}}} = \frac{A_{\text{square}}}{A_{\text{circle}}}$$

$$= \frac{l^2/16}{l^2/4\pi} = \frac{\pi}{4}$$

5. The magnetic moment of current carrying loop

$$M = niA = ni(\pi r^2)$$

Hence the work done in rotating it through 180°

$$W = MB(1 - \cos \theta) = 2MB = 2(ni\pi r^2)B$$

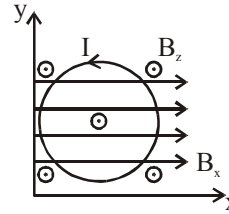
$$= 2 \times (50 \times 2 \times 3.14 \times 16 \times 10^{-4}) \times 0.1 = 0.1 \text{ J}$$

6. Sensitivity $= \frac{NAB}{C}$

7. $\tau_{\text{max}} = NiAB = 1 \times i \times (\pi r^2) \times B$ $\left(2\pi r = L, \Rightarrow r = \frac{L}{2\pi}\right)$

$$\tau_{\text{max}} = \pi \left(\frac{L}{2\pi}\right)^2 B = \frac{L^2 i B}{4\pi}$$

8. $U = -MB \cos \theta$; where θ = Angle between normal to the plane of the coil and direction of magnetic field.



9.

Net torque on the loop must be Balance so

$$i(\pi R^2) B_x = mg R$$

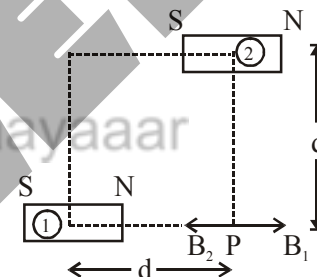
$$\Rightarrow i = \frac{mg}{\pi R B_x}$$

10. $Q = \sigma (2\pi a d)$

$$I = Q \cdot f = \sigma (2\pi a d) f$$

$$B = \frac{\mu_0 I}{2a} = \frac{\mu_0 \sigma (2\pi a d) f}{2a} = \mu_0 \sigma \pi d f$$

11.



$$B_P = B_1 - B_2 = 2 \frac{KM}{d^3} - \frac{KM}{d^3} = \frac{KM}{d^3}$$

where $K = \frac{\mu_0}{4\pi}$ $\therefore B_P = \frac{\mu_0 M}{4\pi d^3}$

12. Apply COME LAW,

$$\Delta kE = - \Delta U$$

$$k_f - k_i = - [U_f - U_i]$$

$$k_f - 0 = U_i - U_f$$

$$k_f = - MB \cos 90^\circ - (-MB \cos 0) = MB$$

13. For dip angle $\tan \theta = \frac{B_v}{B_H} = \frac{1}{1}$ so $\theta = 45^\circ$

& angle of declination = 10°W of N

14. Geographic north will be 15° west of the needle

15. given

$$H = 1600 \text{ A m}^{-1}, B_m = \frac{\phi}{A}$$

$$\Rightarrow B_m = \mu_0(H+I)$$

$$\Rightarrow B_m = \mu_0 H(1+\chi)$$

$$\chi = \frac{B_m}{\mu_0 H} - 1 \approx 596$$

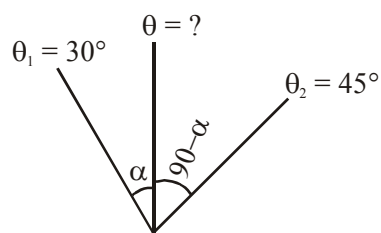
17. $\tan \theta^1 = \frac{\tan \theta}{\cos \alpha}$

$$\begin{aligned} \tan \theta &= \tan \theta^1 \cos \alpha \\ &= \tan 30^\circ \cos 30^\circ \end{aligned}$$

$$\tan \theta = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}$$

$$\theta = \tan^{-1} \left(\frac{1}{2} \right)$$

18.



$$\tan \theta_1 = \frac{\tan \theta}{\cos \alpha} \quad \dots(1)$$

$$\tan \theta_2 = \frac{\tan \theta}{\cos(90 - \alpha)} = \frac{\tan \theta}{\sin \alpha} \quad \dots(2)$$

from equation (1) & (2)

$$\cot^2 \theta_1 + \cot^2 \theta_2 = \cot^2 \theta$$

$$\Rightarrow \cot^2 30 + \cot^2 45 = \cot^2 \theta$$

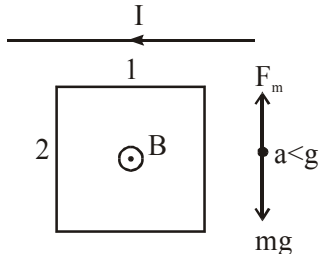
$$\Rightarrow \cot \theta = 2$$

$$\Rightarrow \tan \theta = \frac{1}{2}$$

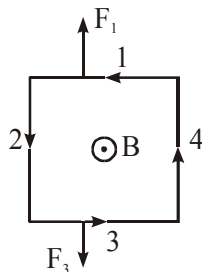
ELECTROMAGNETIC INDUCTION

SOLUTION

1. $\vec{B}$ due to current will be directed upward. As the loop moves downward, flux linked with the loop decreases. So induced current will support the magnetic field to increase the flux.



Hence current will be 'ANTICLOCKWISE'



Force on wire '1'
 $F_1 = I B_1 \ell$ upward
 Force on wire '3'
 $F_3 = I B_2 \ell$ downward
 Net Force is upward as
 $B_1 > B_2$
 So the acceleration
 $a < g$

2. $B = B_0 + \alpha t$
 $A = \pi r^2$
 $\phi = B \cdot A = (B_0 + \alpha t) \pi r^2$
 $\epsilon = \frac{-d\phi}{dt} = -\pi r^2 \frac{d}{dt} (B_0 + \alpha t)$
 $\epsilon = -\pi r^2 \alpha$



Magnetic field due to outer loop is into the plane & increasing with time so the flux is also increasing with time. The induced current will try to decrease the flux so it will produce outward ' $\vec{B}$ '. Hence induced current direction will be 'ANTICLOCKWISE'

4. As the switch is closed, Flux linked with 'B' increases away from eye so the current will be 'ANTICLOCKWISE'. As the switch is opened, flux linked with 'B' decreases, So the current will be 'CLOCKWISE'.

Hence option (4) is correct.

5. $e = NBA\omega \Rightarrow e \propto \omega$

6. $e_0 = -L \frac{d}{dt} (I_0 \sin \omega t)$
 $e = -LI_0 \omega \cos \omega t$
 $e \propto \omega \propto f$

7. $e = \vec{\ell} \cdot (\vec{B} \times \vec{v})$ where $\vec{v} = \hat{i}$ m/sec

$$= 5\hat{j} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 1 & 0 & 0 \end{vmatrix} = 5\hat{j} \cdot (5\hat{j} - 4\hat{k}) = 25 \text{ volt}$$

8. Induced emf = $B \times v \times$ effective length $\perp^r$ to velocity

$$= Bv (\ell_{BD}) = Bv(4R) = 4BvR$$

Since B is uniform & Area is fixed, then 'change in flux is zero in the loop so current flow will be zero.

9. Let at any time t
 flux passing through the coil at time t
 $\phi = BA \cos \theta$

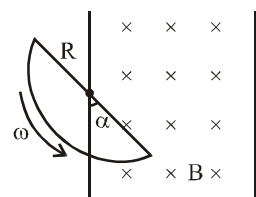
$$\text{Here } \theta = 0^\circ, A = \frac{1}{2} \alpha R^2$$

$$\phi = B \left(\frac{1}{2} \alpha R^2 \right)$$

$$|\text{emf}| = \frac{d\phi}{dt} = \frac{d}{dt} \left(B \frac{1}{2} \alpha R^2 \right) = \frac{1}{2} B R^2 \frac{d\alpha}{dt}$$

$$\text{Here } \frac{d\alpha}{dt} = \omega \text{ (cons.)}$$

$$\text{emf} = \frac{1}{2} B \omega R^2$$



10. Energy stored by Inductor = Energy to be stored in capacitor

$$\Rightarrow \frac{1}{2}LI^2 = \frac{1}{2}CV^2$$

$$\Rightarrow V = I\sqrt{\frac{L}{C}} = 2\sqrt{\frac{2}{4 \times 10^{-6}}} = \sqrt{2} \times 10^3 \text{ V}$$

So order of voltage = 10^3 volts.

11. $P = eI = L \frac{dI}{dt} \cdot I$

$$P = LI \frac{dI}{dt} \quad \text{Since } P = \text{const.}, \quad \frac{dI}{dt} = \text{const.}$$

$$\text{So } LI = \text{const.} \quad \Rightarrow I \propto \frac{1}{L}$$

$$\text{or } \frac{I_1}{I_2} = \frac{L_2}{L_1} = \frac{1}{4}$$

12. $M = K\sqrt{L_1 L_2}$

$$\text{so } K = \frac{M}{\sqrt{L_1 L_2}} = \frac{3}{\sqrt{2 \times 8}} = \frac{3}{4}$$

13. $\phi = B \cdot A = \mu_0 n I \cdot A$

$$\phi = \mu_0 n I_0 \cos \omega t (\pi b^2)$$

$$\varepsilon = -\frac{d\phi}{dt} = \mu_0 n I_0 \pi b^2 \omega \sin \omega t$$

14. $V_A - V_B = L \frac{dI}{dt} + IR$

$$0.5 = L(8) + (0.2 \times 0.5)$$

$$0.4 = L(8)$$

$$L = 0.05 \text{ H}$$

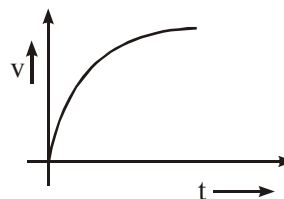
15. At $t = 0$ Inductor behaves as open circuit hence

$$I_1 = 0, I_2 = \frac{E}{R}, \quad I_3 = \frac{E}{2R}$$

$$\text{So } I_2 > I_3 > I_1$$

16. Initially the velocity of bar magnet will increase which will change the flux linked with the copper tube so induced eddy current will oppose the motion of magnet.

So for bar magnet, acceleration decreases with time & velocity increases with time so :



17. $B \text{ due to outer loop} = \frac{\mu_0 I}{4\pi \frac{L}{2}} (8 \sin 45^\circ)$

$$= \frac{\mu_0 I}{\pi L} (2\sqrt{2})$$

Flux linked with smaller loop = $B \cdot A$

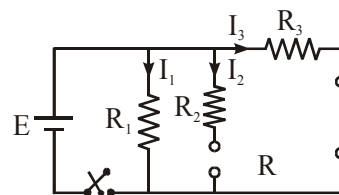
(Assuming B as uniform since $\ell \ll L$)

$$\phi = \frac{\mu_0 I}{\pi L} 2\sqrt{2} \ell^2$$

$$\phi = MI \quad \text{So } M = \frac{\phi}{I}$$

$$\text{So } M = \frac{2\sqrt{2}\mu_0 \ell^2}{\pi L}$$

18. At $t = 0$



$$\text{So } I_1 = \frac{E}{R_1} \quad I_2 = I_3 = 0$$

19. $B = 10 \sin \frac{2\pi t}{3}; \quad A = 0.1$

$$\varepsilon = -\frac{d\phi}{dt} = -A \frac{dB}{dt} = -(0.1)(10) \left(\frac{2\pi}{3} \right) \cos \frac{2\pi}{3} t$$

$$|\varepsilon| = \frac{2\pi}{3} \cos \left(\frac{2\pi}{3} t \right) \quad \text{at } t = 0.5 \text{ sec.}$$

$$|\varepsilon| = \frac{2\pi}{3} \cos \left(\frac{\pi}{3} \right) = \frac{\pi}{3} \text{ volts}$$

20. As current across Resistor 'R' increases, Magnetic field linked with loop 2 increases in the direction outward, $\perp^r$ to the plane of paper so the induced current will decrease the flux. & It will flow in clockwise direction to oppose the initial magnetic field.

$$21. \quad i = i_0[1 - e^{-\frac{t}{\lambda}}]$$

$$i = \frac{i_0}{4}$$

$$\Rightarrow \frac{i_0}{4} = i_0 \left[1 - e^{-\frac{t}{\lambda}} \right]$$

$$\Rightarrow \frac{1}{4} = 1 - e^{-\frac{4}{\lambda}}$$

$$\Rightarrow e^{-\frac{4}{\lambda}} = \frac{3}{4}$$

$$\Rightarrow e^{4/\lambda} = \frac{4}{3}$$

$$\Rightarrow \frac{4}{\lambda} = \log_e \left(\frac{4}{3} \right)$$

$$\Rightarrow \lambda = \frac{4}{\log_e \left(\frac{4}{3} \right)}$$

22. Energy per unit volume

$$U_V = \frac{B^2}{2\mu_0}$$

$$U = \frac{B^2}{2\mu_0} \times V \quad (\text{total energy stored})$$

$$= \left(\frac{\mu_0 i}{2r} \right)^2 \times \frac{V}{2\mu_0}$$

$$\Rightarrow \frac{\mu_0 i^2 V}{8r^2}$$

$$\Rightarrow \frac{\pi}{2} \times 10^{-16} \text{ J}$$

23. Heat $H = \int i^2 R dt$

$$i = \frac{1}{R} \frac{d\phi}{dt} \Rightarrow \frac{1}{4} \times \frac{d}{dt} (2t^2 + 9)$$

$$[i = t] \text{ Amp.}$$

$$H = \int_0^3 t^2 \cdot 4 dt \Rightarrow \left[\frac{4t^3}{3} \right]_0^3 = 36 \text{ J}$$

$$1. \quad V_{\text{rms}} = \sqrt{\bar{V}^2} \Rightarrow V_{\text{rms}}^2 = \bar{V}^2 = \frac{\int_{t_1}^{t_2} V^2 dt}{t_2 - t_1}$$

$$\therefore V_{\text{rms}}^2 = \frac{\int_0^1 (V_0 t^{3/2})^2 dt}{1-0} = V_0^2 \int_0^1 t^3 dt$$

$$= V_0^2 \left[\frac{t^4}{4} \right]_0^1 = V_0^2 \left[\frac{1}{4} - \frac{0}{4} \right] = \frac{V_0^2}{4}$$

$$\text{therefore, } V_{\text{rms}} = \frac{V_0}{2}$$

$$3. \quad \text{using KCL, } I_3 = I_1 + I_2$$

$$I_3 = 3A \sin \omega t + 4A \cos \omega t$$

$$I_3 = R \sin (\omega t + \theta)$$

$$\text{where } R = \sqrt{(3A)^2 + (4A)^2} = 5A$$

$$\tan \theta = \frac{4}{3} \Rightarrow \theta = 53^\circ$$

$$4. \quad \text{Heat produced by ac} = 4 \text{ (heat produced by steady current)}$$

$$i_{\text{rms}}^2 R = 4(i_{\text{dc}}^2 R)$$

$$i_{\text{rms}} = 2i_{\text{dc}} = 2 \times 2A = 4A$$

$$\therefore i_0 = \sqrt{2} i_{\text{rms}} = 4\sqrt{2} A = 5.656 A$$

$$5. \quad i = 2 + 3 \sin \omega t$$

$$i_{\text{rms}} = \sqrt{2^2 + 3^2/2} = \sqrt{17/2}$$

$$6. \quad \text{For the lamp } R = \frac{V_{\text{dc}}}{i_{\text{dc}}} = \frac{100}{10}$$

$$\Rightarrow R = 10 \Omega$$

$$\text{with ac, } Z = \frac{V}{i} = \frac{200}{10} = 20\Omega$$

$$\therefore Z = \sqrt{R^2 + X_L^2}$$

$$\Rightarrow X_L = \sqrt{Z^2 - R^2}$$

$$\omega L = \sqrt{400 - 100} = 10\sqrt{3} \Omega$$

$$L = \frac{10\sqrt{3}}{2\pi(50)} = \frac{17.32}{3.14} \times 10^{-2} \text{ H} = 5.5 \times 10^{-2} \text{ H}$$

$$7. \quad \text{With dc source}$$

$$R = \frac{V}{I} = \frac{12V}{2A} = 6\Omega$$

with ac source

$$Z = \frac{V}{i} = \frac{12V}{1A} = 12\Omega$$

$$\therefore Z^2 = R^2 + X_L^2$$

$$\Rightarrow X_L = \sqrt{12^2 - 6^2} = 6\sqrt{3} \Omega$$

$$\text{use } 2\pi fL = X_L \Rightarrow L = 33 \text{ mH}$$

$$8. \quad X \text{ is resistance } (\because \text{phase difference} = 0)$$

$$\therefore R = \frac{V}{i} = \frac{220}{0.5} = 440\Omega$$

$$Y \text{ is capacitor } (\because \text{phase difference} = -\frac{\pi}{2})$$

$$\therefore X_C = \frac{V}{i} = \frac{220}{0.5} = 440 \Omega$$

on connecting X and Y in series

$$Z = \sqrt{R^2 + X_C^2} = 440\sqrt{2}\Omega$$

$$\therefore i = \frac{V}{Z} = \frac{220}{440\sqrt{2}} = \frac{0.5}{\sqrt{2}} A = 0.35A$$

$$9. \quad V = \sqrt{(120)^2 + (90)^2} = 150 \text{ volt}$$

$$\therefore Z = \frac{V}{i} = \frac{150}{3} = 50\Omega$$

$$10. \quad i = 10 + 40 \cos \omega t$$

$$\therefore i_{\text{rms}} = \sqrt{(10)^2 + (40)^2/2}$$

$$12. \quad \text{Currents in same phase in series combination}$$

$$13. \quad E = E_0 \sin \omega t$$

$$\omega = 75\pi$$

$$2\pi f = 75\pi$$

$$f = \frac{75}{2} \text{ Hz}$$

In one cycle ac current becomes zero twice

$\therefore$ 75 times zero is one second

$$1. \quad V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(40)^2 + (60 - 30)^2}$$

$$= 50 \text{ volt}$$

$$2. \quad \text{By phasor}$$

$$V_{LC} = V_L - V_C$$

$$= 50V - 50V$$

$$= 0V$$

$$3. \quad \because X_L = X_C \Rightarrow \text{Resonance}$$

$$\text{Reading of voltmeter} = V_L - V_C = 0V$$

$$\text{and } i = \frac{V}{R} = \frac{240}{30} = 8A$$

$$5. \quad V_R = \sqrt{(50)^2 - (100 - 60)^2} = 30V$$

$$\therefore i = \frac{V_R}{R} = \frac{30V}{10\Omega} = 3A$$

6. In case of Resonance

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8 \times 10^{-3} \times 20 \times 10^{-6}}} = 2500 \text{ rad/s}$$

$$i = \frac{V}{R} = \frac{220}{44} A = 5A$$

$$\text{peak value} = \sqrt{2} i_{\text{rms}} = 5\sqrt{2} A$$

$$8. \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Now At $f = 0$

$$X_L = 0 \text{ \& } X_C = \infty \Rightarrow Z = \infty$$

$$\Rightarrow i = 0$$

At $f = \infty$

$$X_C = 0 \text{ \& } X_L = \infty$$

$$\Rightarrow Z = \infty$$

$$i = 0$$

$$10. \quad \text{With sinusoidal wave } P = i_{\text{rms}}^2 R$$

$$= \frac{i_0^2 R}{2} = W \dots (1)$$

with equal wave

$$P = i_{\text{rms}}^2 R = i_0^2 R = 2W$$

11. Here $I_0 = 1.0 A$
Bandwidth applies to frequencies when

$$I = \frac{I_0}{\sqrt{2}} = 0.707 A$$

$$\Delta\omega = \omega_2 - \omega_1 = (1.4 - 0.8) = 0.6 \text{ rad s}^{-1}$$

12. since, $R = X_C = X_L$ [$\because$ of equal voltage]
When capacitor is short circuited

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{10}{\sqrt{2}R} \quad [X_L = R]$$

$$\therefore V_L = IX_L = IR = \frac{10}{\sqrt{2}R} \times R = \frac{10}{\sqrt{2}} V$$

EM WAVE : RACE # 01

SOLUTIONS

1. $\frac{E_0}{B_0} = C = \frac{\omega}{K}$ (in free space)

3. $I_D = A\epsilon_0 \left(\frac{dE}{dt} \right) = c \frac{dv}{dt}$

4. Due to electric and magnetic field.

5. Wavelength $\lambda = \frac{c}{v} = 2\pi c \sqrt{LC}$
 $= 2 \times 3.14 \times 3 \times 10^8 \sqrt{400 \times 10^{-12} \times 100 \times 10^{-6}}$
 $= 377 \text{ m}$

7. Zero

10. $B = \frac{E}{c} = \frac{1}{3 \times 10^8} = 3.33 \times 10^{-9} \text{ T}$. Direction $\vec{B}$

can be checked by $\vec{E} \times \vec{B}$ = Direction of propagation of EM wave.

11. $\frac{E_{\max}}{B_{\max}} = v$ and $B_{\max} = \mu_0 H_{\max}$

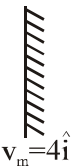
$$\Rightarrow H_{\max} = \frac{E_{\max}}{\mu_0 v} = \frac{100}{(4\pi \times 10^{-7}) \left(\frac{10^8}{2} \right)} = \frac{5}{\pi}$$

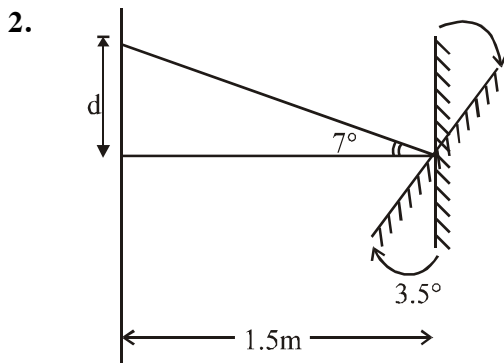
12. $\vec{E} \times \vec{B}$

13. $F = \frac{IA}{c}$
 $= \frac{1400 \times 1.5}{3 \times 10^8} = 7 \times 10^{-6} \text{ N}$

RAY OPTICS (PLANE MIRROR) : RACE # 01

SOLUTIONS

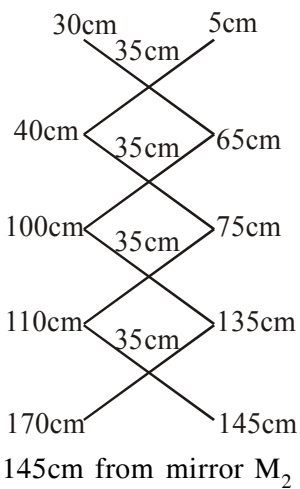
1. $\vec{v}_0 = 3\hat{i}$

 for component of velocity perpendicular to mirror
 $\vec{v}_I = 2\vec{v}_m - \vec{v}_0 = 8\hat{i} - 3\hat{i} = 5\hat{i}$
 other component will not change
 $\Rightarrow V_I = 5\hat{i} + 4\hat{j} + 5\hat{k}$



If mirror rotated by θ , then reflected ray rotates by
 $2\theta = 2 \times 3.5 = 7^\circ$
 for θ small
 $\theta \approx \tan \theta$

$$\frac{d}{1.5} = \frac{7\pi}{180}$$

$$d = \frac{7 \times 3.14 \times 1.5}{180} = 0.1831\text{m} = 18.31\text{ cm}$$

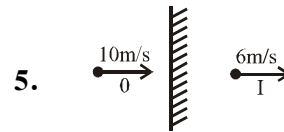
3. 

4. $\delta = 2\pi - 2\theta = 2\pi - 2\alpha$

$$\Rightarrow 2\pi - \frac{5\pi}{6} = 2\alpha$$

$$\Rightarrow \frac{7\pi}{6} = 2\alpha$$

$$\Rightarrow \alpha = \frac{7\pi}{12}$$



$$\Rightarrow \vec{V}_I = 2\vec{V}_m - \vec{V}_0$$

$$\vec{V}_0 = 10\hat{i}$$

$$\vec{V}_I = 6\hat{i}$$

$$\vec{V}_m = \frac{10\hat{i} + 6\hat{i}}{2} = 8\hat{i} \text{ m/s}$$

$$\vec{V}_m = 8\text{m/s, away from the object}$$

6. The walls act as two mirrors inclined to each other at 90° and so they will form 3 images of the person. Now these images and person will act as object for the ceiling mirror and ceiling mirror will form 4 images. Therefore total number of images formed = $3 + 4 = 7$

RAY OPTICS (SPHERICAL MIRROR) : RACE # 02

SOLUTION

1. For Real image

$$m = -2$$

$$m = \frac{f}{f - u}$$

$$-2 = \frac{-50}{-50 - u}$$

$$u = -75 \text{ cm} \quad \{-ve \text{ indicates real object}\}$$

2. For real image $m = -3$

$$m = \frac{f}{f - u}$$

$$-3 = \frac{f}{f - (-20)}$$

$$-3 = \frac{f}{f + 20}$$

$$-3f - 60 = f \Rightarrow f = -15 \text{ cm.}$$

($f = -ve \Rightarrow$ concave mirror)

3. $\frac{-v}{u} = \frac{f}{f - u}$

$$\frac{-v}{-20} = \frac{10}{10 - (-20)}$$

$$v = \frac{20 \times 10}{30} \quad \text{or} \quad v = \frac{20}{3} \text{ cm}$$

v is (+) ve, so, virtual image

4. $\frac{h_2}{h_1} = \frac{f}{f - u}$

$$\frac{h_2}{2.5} = \frac{-15}{-15 - (-10)}$$

$$h_2 = \frac{2.5(-15)}{-5}$$

$$h_2 = 7.5 \text{ cm}$$

5. $m = \frac{f}{f - u} = \frac{-18}{-18 - (-27)} \Rightarrow m = \frac{-18}{9} = -2$

$$m = \frac{f - v}{f} \Rightarrow -2 = \frac{-18 - v}{-18} \Rightarrow v = -54 \text{ cm}$$

$$\text{therefore } h_2 = 2 \times 2.5 = 5 \text{ cm}$$

6. $f = +15 \text{ cm}$, $h_1 = 4.5 \text{ cm}$, $u = -12 \text{ cm}$

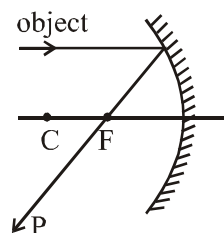
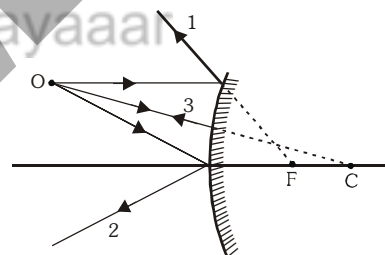
$$\frac{-v}{u} = \frac{f}{f - u}$$

$$\frac{-v}{-12} = \frac{15}{15 - (-12)}$$

$$v = \frac{12(15)}{27} = 6.7 \text{ cm}$$

$$m = \frac{-f}{f - u} = \frac{15}{27} = \frac{5}{9}$$

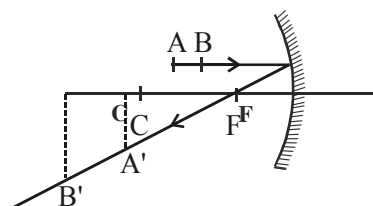
7. Correct Ray diagram is :



- 8.

9. Aberrations can occur for virtual image also.

- 10.



11. $\frac{1}{-0.5} + \frac{1}{0.2} = \frac{2}{R} \Rightarrow \frac{2}{R} = 3; R = 0.667 \text{ m}$

12. Distance of object is equal to focal length

So, $u = \pm f$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{\pm f} = \frac{1}{f}$$

$$v = 2f, \infty$$

13. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating, we get $dv = -\frac{v^2}{u^2} du$

$$dv = -\left(\frac{-12}{-60}\right)^2 [1 \text{ mm}]$$

$$dv = -\frac{1}{25} \text{ mm}$$

[$\because du = 1 \text{ mm}$; sign of du is +ve because it is shifted in +ve direction defined by sign convention]

Tg: @Chalnaayaaar

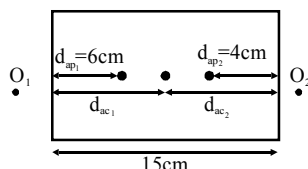
1. Stars appear to twinkle due to fluctuations of refractive index of earth's atmosphere.

2. Object is in rare medium & observed from denser

$$d_{app} = \mu d_{ac}$$

$$d_{ac} = \frac{d_{app}}{\mu} = \frac{30}{4/3} \Rightarrow \frac{90}{4} \Rightarrow 22.5 \text{ cm}$$

3.



$$\therefore d_{ap} = \frac{d_{ac}}{\mu}$$

$$d_{ac} = d_{ap} \times \mu$$

$$\therefore d_{ac1} = 6 \times \mu$$

$$\& d_{ac2} = d_{ap2} \times \mu = 4 \times \mu$$

$$\therefore d_{ac1} + d_{ac2} = 15$$

$$\Rightarrow 6\mu + 4\mu = 15 \Rightarrow \mu = 1.5$$

4. glass thickness = 4 mm
and $\mu = 3$ (given)

$$\therefore \text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{4 \times 10^{-3}}{v}$$

$$\text{Time} = \frac{4 \times 10^{-3}}{(c/\mu)} = \frac{4 \times 10^{-3} \times \mu}{c}$$

$$\text{Time} = \frac{4 \times 10^{-3} \times 3}{3 \times 10^8} = 4 \times 10^{-11} \text{ second}$$

$$5. x = d_{ac} \left(1 - \frac{1}{\mu}\right) = 15 \left(1 - \frac{1}{1.5}\right)$$

$$x = \frac{15}{3} = 5 \text{ cm}$$

$$6. \sin \theta_c = \frac{\mu_R}{\mu_D}$$

$$\sin \theta_c = \frac{v_D}{v_R}$$

In Denser medium speed is slow (2.2×10^8)
whereas in rare medium speed is high (2.4×10^8)

$$\sin \theta_c = \frac{2.2 \times 10^8}{2.4 \times 10^8}$$

$$\theta_c = \sin^{-1} \left(\frac{11}{12} \right)$$

7. From (a)

$$\mu_{air} \sin 60^\circ = \mu_{glass} \sin 45^\circ$$

$${}_a\mu_g = \sqrt{\frac{3}{2}} = \mu_g$$

from (b)

$$\mu_{air} \sin 60^\circ = \mu_{water} \sin 30^\circ$$

$${}_a\mu_w = \sqrt{3} = \mu_w$$

Now, from (c)

$$\mu_{water} \sin 45^\circ = \mu_{glass} \sin r$$

$$\Rightarrow \sqrt{3} \sin 45^\circ = \sqrt{\frac{3}{2}} \sin r$$

$$\sin r = 1 \Rightarrow r = 90^\circ$$

8. $n_{quartz} = 2, n_{glycerine} = 4/3$

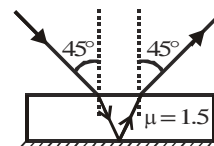
$$\text{Shift} = t \left(1 - \frac{1}{n_{rel}} \right) = 18 \left(1 - \frac{1}{\frac{2}{\frac{4}{3} \times 3}} \right) = 6 \text{ cm}$$

9. Glass slab produces zero deviation, so angle of divergence does not change.

10. Height of air = Apparent depth of water

$$21 - x = \frac{x}{4/3} \Rightarrow x = 12 \text{ cm}$$

11.



$$\delta = 180 - 2i = 90^\circ$$

Parallel slab don't produce extra deviation.

12. $n_1 \sin 45^\circ = n_3 \sin \theta$

$$\Rightarrow (1) \left(\frac{1}{\sqrt{2}} \right) = \sqrt{2} \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

13. $1 \times \sin 45^\circ = \sqrt{2} \sin \gamma \Rightarrow \gamma = 30^\circ$

distance travelled $AB \cos 30^\circ = 2$

$$\Rightarrow AB = \frac{4}{\sqrt{3}} \text{ m and speed} = \frac{3 \times 10^8}{\sqrt{2}}$$

$$\Rightarrow t = \frac{AB}{v} = \frac{4\sqrt{2}}{3\sqrt{3}} \times 10^{-8} \text{ s}$$

14. $\mu_{\text{rarer}} = 1.5, i = 60^\circ, r = 45^\circ$

$\mu_{\text{denser}} = ?$

$\mu_{\text{rarer}} \sin i = \mu_{\text{denser}} \sin r$

$\Rightarrow 1.5 \times \sin 60^\circ = \mu_{\text{denser}} \sin 45^\circ$

$\Rightarrow \mu_{\text{denser}} = \frac{1.5 \times \sin 60^\circ}{\sin 45^\circ}$

$\Rightarrow \mu_{\text{denser}} = \frac{1.5 \times (\sqrt{3}/2)}{(1/\sqrt{2})} = \frac{1.5 \times \sqrt{3} \times \sqrt{2}}{2} = 1.84$

15. $1 \times \sin 60^\circ = \mu_{\text{oil}} \times \sin 30^\circ$

$\mu_{\text{oil}} = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

16. Given :

Refractive index of glass $\mu_g = \sqrt{2}$

angle of incidence $i = 45^\circ$

$\mu_{\text{air}} = 1$

$\mu_g \sin i = \mu_{\text{air}} \sin r$

$\sqrt{2} \times \sin 45^\circ = (1) \times \sin r$

$\sqrt{2} \times \frac{1}{\sqrt{2}} = \sin r$

$\sin r = 1 \Rightarrow r = 90^\circ$

17. $\sin \theta_c = \frac{\mu_R}{\mu_D}$

$\sin \theta_c = \frac{\left(\frac{4}{3}\right)}{\left(\frac{5}{3}\right)} = \frac{4}{5} \Rightarrow \theta_c = \sin^{-1}\left(\frac{4}{5}\right)$

18. $\therefore \sin \theta_c = \frac{1}{\mu} \text{ \& } \mu \propto \frac{1}{\lambda}$

$\therefore \theta_c = \theta$

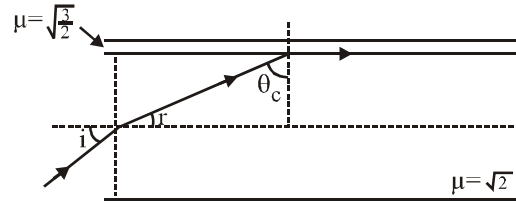
$\therefore \sin \theta \propto \lambda \Rightarrow$ Less for yellow colour.

19. $h = 80 \text{ cm} \quad \mu = 1.33$

$r = \frac{h}{\sqrt{\mu^2 - 1}} = \frac{80 \text{ cm}}{\sqrt{(1.33)^2 - 1}} = 0.90 \text{ m} = 0.90 \text{ m}$

Area = $\pi r^2 = 3.14 \times (0.90)^2 = 2.54 \text{ m}^2$

20.



Ist method

$\sin \theta_c = \frac{\mu_R}{\mu_D} = \frac{\sqrt{3}/2}{\sqrt{2}} \quad \sin \theta_c = \frac{\sqrt{3}}{2}$

$\theta_c = 60^\circ$

$r = 90^\circ - \theta_c = 90^\circ - 60^\circ$

$r = 30^\circ$

Now, $1 \times \sin i = \sqrt{2} \times \sin(30^\circ)$

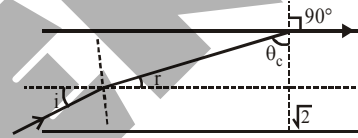
$i = 45^\circ$

IInd method

$\sin i = \sqrt{n_1^2 - n_2^2} = \sqrt{(\sqrt{2})^2 - \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\sqrt{2}}$

$\Rightarrow i = 45^\circ$

21.



$\sin \theta_c = \frac{\mu_R}{\mu_D} = \frac{1}{\sqrt{2}}$

$\theta_c = 45^\circ$

$r = 90 - \theta_c$

$r = 90 - 45^\circ = 45^\circ$

Now,

$1 \times \sin i = \sqrt{2} \times \sin 45^\circ$

$\Rightarrow i = 90$

Or

$\sin i = \sqrt{n_1^2 - n_2^2} = \sqrt{(\sqrt{2})^2 - (1)^2}$

$\sin i = 1$

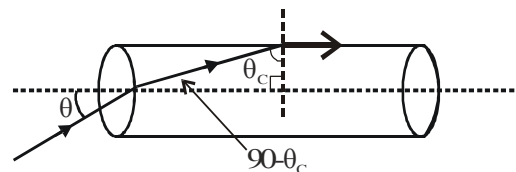
or $i = 90^\circ$

22. $1 \times \sin \theta = \mu \sin (90 - \theta_c)$

$\sin \theta = \frac{2}{\sqrt{3}} \cos \theta_c = \frac{2}{\sqrt{3}} \times \frac{1}{2}$

$\sin \theta = \frac{1}{\sqrt{3}} \quad \left(\sin \theta_c = \frac{\sqrt{3}}{2} \Rightarrow \theta_c = 60^\circ \right)$

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

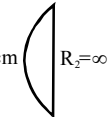


RAY OPTICS (Lenses) : RACE # 04

SOLUTION

1. $\therefore \frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$

lens is plano-convex lens, means

$\therefore \frac{1}{f} = (1.6 - 1) \left[\frac{1}{18} - \frac{1}{\infty} \right]$ 

$\frac{1}{f} = 0.6 \times \frac{1}{18}$ $f = 30$ cm

2. When $\mu_{\text{lens}} < \mu_{\text{medium}}$
then lens behaves as a diverging lens.

3. $\frac{v}{u} = \frac{f}{f + u}$

$\frac{v}{-10} = \frac{12}{12 - 10}$

$v = \frac{(-10)(12)}{2}$

$v = -60$ cm (on same side of object)

4. Given

$f = -20$ cm & $u = -10$ cm

$\therefore m = \frac{f}{f + u} = \frac{-20}{-20 - 10} = \frac{-20}{-30}$

$m = \frac{2}{3} = +0.667$

5. $\therefore P_{\text{com}} = 2P_L + P_m$

Here,

$P_L = \frac{1}{f_L} = (1.5 - 1) \left[\frac{1}{10} - \frac{1}{\infty} \right]$, $P_m = 0$

$\Rightarrow \frac{1}{f_L} = \frac{0.5}{10} \Rightarrow f_L = \frac{10}{0.5} = 20$ cm

$P = \frac{2}{20} + \frac{1}{\infty}$

$f_{\text{com}} = -\frac{1}{P_{\text{com}}} = -10$ cm (concave mirror)

6. Given

$u = -1.2$ m, $v = 6 - 1.2 = 4.8$ m

$m = \frac{v}{u} = \frac{+4.8}{-1.2} = -4$

7. $\therefore \frac{1}{f} = \frac{1}{f_{\text{convex}}} + \frac{1}{f_{\text{concave}}}$

$\frac{1}{f} = \frac{1}{40} + \frac{1}{(-25)}$

$\frac{1}{f} = \frac{1}{40} - \frac{1}{25} = \frac{5-8}{200} = \frac{-3}{200}$

$f = -\frac{200}{3}$ cm

$\therefore P = \frac{100}{f(\text{cm})} = \frac{100}{\left(-\frac{200}{3}\right)} = -\frac{300}{200} = -\frac{3}{2}$ D

- 8.



$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{f} = \frac{1}{20} + \frac{1}{20} \Rightarrow f = 10$ cm



$u = -20$ cm

$f = 10$

object is present at 2F So image will be real, inverted and same size.

9. Focal length of this silvered lens

$P_{\text{com}} = 2P_L + P_m$

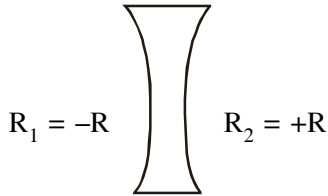
$-\frac{1}{f} = \frac{2}{f_L} + \frac{1}{f_m}$ $\frac{1}{f} = \frac{2}{20} + \frac{1}{\infty} = \frac{1}{10}$

$f = -10$ cm



$$10. \quad \therefore \frac{1}{f} = (\mu_{\ell} - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$= \left[\frac{1.5}{1.75} - 1 \right] \left[-\frac{1}{R} - \frac{1}{(+R)} \right]$$



$$\frac{1}{f} = \left(-\frac{1}{7} \right) \times \left(-\frac{2}{R} \right) = \frac{2}{7R} \Rightarrow f = 3.5 R$$

$\therefore f$ is positive so (not depend on medium)
converging lens

$$11. \quad f_{\text{mirror}} = \frac{R}{2}$$

$$\frac{1}{f_{\text{lens}}} = \left(\frac{\mu_L}{\mu_M} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$12. \quad m = \frac{\text{image size}}{\text{object size}} \quad (\text{As image is real})$$

$$- \mu = \frac{f}{f + u} \quad \text{therefore } u = \frac{(-\mu - 1)f}{\mu}$$

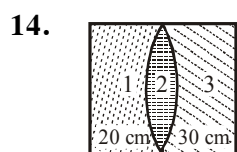
$$\Rightarrow \text{distance} = |u| = \frac{(\mu + 1)f}{\mu}$$

$$13. \quad \therefore \frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{30} = (\mu - 1) \left[\frac{1}{10} - \frac{1}{\infty} \right]$$

$$\frac{1}{30} = (\mu - 1) \times \frac{1}{10}$$

$$\frac{1}{3} = (\mu - 1) \Rightarrow \mu = \frac{1}{3} + 1 = \frac{4}{3} = 1.33$$



focal length of the system

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Here,

$$\text{for lens 1 ; } \frac{1}{f_1} = \left[\frac{3}{2} - 1 \right] \left[\frac{1}{-\infty} - \frac{1}{20} \right] = -\frac{1}{40}$$

$$\text{for lens 2 ; } \frac{1}{f_2} = \left[\frac{4}{3} - 1 \right] \left[\frac{1}{20} - \frac{1}{(-30)} \right] = +\frac{5}{180}$$

$$\text{for lens 3 ; } \frac{1}{f_3} = \left[\frac{3}{2} - 1 \right] \left[-\frac{1}{30} - \frac{1}{\infty} \right] = -\frac{1}{60}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = -\frac{1}{40} + \frac{5}{180} - \frac{1}{60}$$

$$F = -72 \text{ cm}$$

$$15. \quad \frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

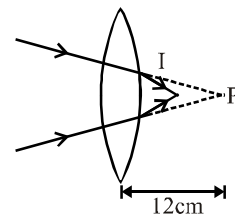
$$\frac{1}{20} = (1.55 - 1) \left[\frac{1}{R} - \left(-\frac{1}{R} \right) \right]$$

$$R = 22 \text{ cm}$$

$$16. \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{(+12)} = \frac{1}{20}$$

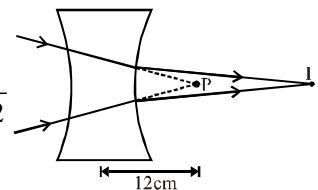
$$v = +7.5 \text{ cm}$$



$$17. \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = -\frac{1}{16} + \frac{1}{12}$$

$$v = +48 \text{ cm}$$



$$18. \quad h_1 = 3 \text{ cm} \quad u = -14 \text{ cm} \quad f = -21 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-14)} = -\frac{1}{21}$$

$$v = -8.4 \text{ cm}$$

$$\frac{h_2}{h_1} = \frac{v}{u} \Rightarrow \frac{h_2}{3} = \frac{-8.4}{-14} \Rightarrow h_2 = 1.8 \text{ cm}$$

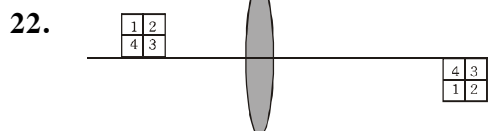
$$19. \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{(-20)}$$

$$\Rightarrow f = -60 \text{ cm}$$

20. System of negative power so diverging

21. Focal length is positive,

No matter in what orientation the lens is placed, as it is thick in the middle so it will converge the rays as surrounding medium is air.

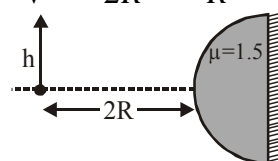


24. A water drop in air behaves as converging lens

25. Here, three optical phenomena take place – first refraction, then reflection, and finally refraction.

For refraction at 1st

$$\frac{1.5}{v} - \frac{1}{-2R} = \frac{1.5-1}{R} \Rightarrow \frac{1.5}{v} = 0 \Rightarrow v = \infty$$



i.e., rays after refraction get parallel to axis and strike the mirror normally, get retraced back and the final image is formed at the same place where the object is and of the same size. Image would be real.

$$26. \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\mu_1 = 1.5, \mu_2 = 1, R = -5, u = -3 \text{ cm}$$

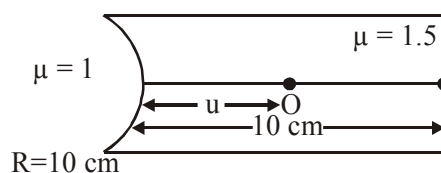
$$\frac{1}{v} - \frac{(1.5)}{-3} = \frac{1-1.5}{-5} \Rightarrow v = -2.5 \text{ cm}$$

air bubble will appear at a distance 2.5 cm from the curved surface inside the glass.

$$27. \mu_1 = 1.5, v = -10 \text{ cm}$$

$$\mu_2 = 1, R = +10 \text{ cm}$$

$$\frac{-1}{10} - \frac{1.5}{u} = \frac{1-1.5}{10} \Rightarrow u = -30 \text{ cm}$$



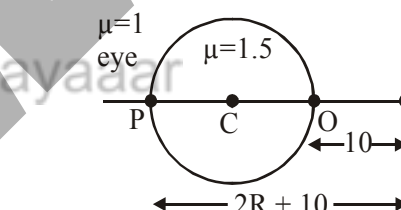
air bubble is actually located at a distance 30 cm from the surface and inside glass.

$$28. \mu_1 = 1.5, \mu_2 = 1, u = -2R$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

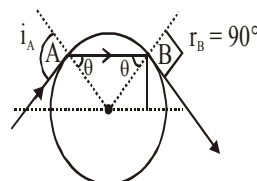
$$\frac{1}{v} - \frac{1.5}{-2R} = \frac{1-1.5}{-R}$$

$$\Rightarrow v = -4R$$



If in given that the image of mark is at a distance 10 cm from object, so $4R = 2R + 10$
 $\Rightarrow R = 5 \text{ cm}$

29.



Angle of refraction at A = angle of incident at B
 Angle of incidence at A = Angle of refraction at B which is 90° for grazing emergence

RAY OPTICS (PRISM) : RACE # 05

SOLUTION

1. $A = 60^\circ$ & $\delta_m = 30^\circ$ (given) $\therefore \delta_m = 2i - A$

$\therefore 30^\circ = 2i - 60^\circ \Rightarrow i = 45^\circ$

2. $\mu = 1.5$ & $A = 60^\circ$ (for equilateral prism) (given)

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin(A/2)}$$

$$\Rightarrow 1.5 = \frac{\sin\left(\frac{60^\circ + \delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$\Rightarrow 1.5 \times \sin(30^\circ) = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\Rightarrow 1.5 \times \frac{1}{2} = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\Rightarrow 0.75 = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\frac{60^\circ + \delta_m}{2} = \sin^{-1}(0.75)$$

$$\Rightarrow \frac{60^\circ + \delta_m}{2} = 49^\circ$$

$$\Rightarrow 60^\circ + \delta_m = 2 \times 49^\circ$$

$$\Rightarrow \delta_m = 2 \times 49^\circ - 60^\circ = 38^\circ$$

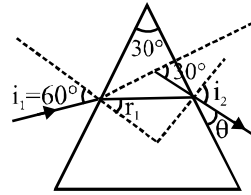
3. $\delta_m = A$ & $\mu = \sqrt{3}$ (given)

$$\therefore \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin(A/2)} \Rightarrow \sqrt{3} = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin(A/2)}$$

$$\sqrt{3} = \frac{2\sin(A/2)\cos(A/2)}{\sin(A/2)} \Rightarrow \frac{\sqrt{3}}{2} = \cos(A/2)$$

$$\frac{A}{2} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ \Rightarrow A = 2 \times 30^\circ = 60^\circ$$

4.



$$\therefore \delta = i_1 + i_2 - A$$

$$30^\circ = 60^\circ + i_2 - 30^\circ$$

$$i_2 = 0^\circ$$

Means, emergent ray goes along the normal & it will make angle with the second face of the prism is $\theta = 90^\circ - 0^\circ = 90^\circ$

5. $\therefore i_2 = 0^\circ$

therefore $r_2 = 0$

6. $\therefore A = r_1 + r_2$

$$30^\circ = r_1 + 0^\circ \quad r_1 = 30^\circ$$

7. From Snell's rule $1 \times \sin i_1 = \mu \times \sin r_1$

$$\sin 60^\circ = \mu \times \sin r_1 \Rightarrow \sin 60^\circ = \mu \times \sin 30^\circ$$

$$\frac{\sqrt{3}}{2} = \mu \times \frac{1}{2} \Rightarrow \mu = \sqrt{3} = 1.732$$

8. Given -

$$A = 5^\circ \text{ & } \mu = 1.54 \quad \text{for prism } P_1$$

$$\mu' = 1.92 \text{ & } A' = ? \quad \text{for prism } P_2$$

for dispersion without deviation

$$(\mu - 1)A + (\mu' - 1)A' = 0$$

$$\Rightarrow (1.54 - 1) \times 5^\circ = -(1.92 - 1)A'$$

$$\Rightarrow A' = -2.9^\circ$$

9. Given

$$i_1 = 45^\circ, A = 60^\circ, \delta_m = 55^\circ, i_2 = ?$$

$$\therefore \delta_m = i_1 + i_2 - A$$

$$\Rightarrow 55^\circ = 45^\circ + i_2 - 60^\circ$$

$$\Rightarrow i_2 = 55^\circ + 60^\circ - 45^\circ = 70^\circ$$

10. Given

$$\text{for equilateral prism } A = 60^\circ$$

$$\text{& } i = 35^\circ$$

$$\therefore \delta_{\min} = 2i - A = 2 \times 35^\circ - 60^\circ$$

$$\Rightarrow \delta_{\min} = 10^\circ$$

$$11. \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \mu = \frac{\sin\left(\frac{60^\circ + 30^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} = \frac{\sin 45^\circ}{\sin 30^\circ}$$

$$\Rightarrow \mu = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} = 1.414$$

$$12. \omega = \frac{\mu_v - \mu_R}{\mu_y - 1}$$

$$\mu_y = \frac{\mu_v + \mu_R}{2}$$

$$13. \theta = (\mu_v - \mu_R) A = (1.68 - 1.56) \times 12^\circ = 1.44^\circ$$

14. For retracing of light, light will fall normally on the mirror.

By applying Snell's law $\mu_1 \sin i = \mu_2 \sin r$

$$1 \times \sin 45^\circ = \mu \sin 30^\circ$$

$$\mu = \sqrt{2}$$

$$15. \delta = i + e - A \Rightarrow A = 45^\circ \quad (i = 15^\circ; e = 60^\circ)$$

$$16. \delta_m = 2i - A = 2 \times 40^\circ - 60^\circ = 20^\circ$$

$$17. \text{Since } \theta_1 = \theta_2 \quad \{\text{achromatism}\}$$

and given $\omega_1 > \omega_2$

$$\Rightarrow \frac{\theta_1}{\delta_1} > \frac{\theta_2}{\delta_2} \Rightarrow \delta_2 > \delta_1 \quad \{\text{as } \theta_1 = \theta_2\}$$

Net deviation will be

$$\delta_2 - \delta_1 = \text{ACW.}$$

Tg: @Chalnaayaaar

1. $m = 1 + \frac{D}{f}$, $m = 1 + \frac{25}{5} = 6$

2. $\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

image is virtual, therefore

$$v = -25 \text{ cm}$$

$$\frac{1}{-25} - \frac{1}{u} = \frac{1}{5} \quad \text{or} \quad \boxed{\frac{v}{u} = \frac{f}{f+u}}$$

$$\Rightarrow \frac{1}{u} = -\frac{1}{25} - \frac{1}{5}$$

$$\Rightarrow \frac{1}{u} = -\frac{6}{25} \Rightarrow u = -\frac{25}{6} = -4.16 \text{ cm}$$

3. $f_0 = 1 \text{ cm}$ & $f_e = 2 \text{ cm}$
 $L = 12 \text{ cm}$ & $v_e = D = -25 \text{ cm}$ (given)
 for eyepiece

$$\therefore \frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{(-25)} - \frac{1}{u_e} = \frac{1}{2}$$

$$\frac{1}{u_e} = -\frac{1}{25} - \frac{1}{2} \quad \text{or} \quad \boxed{\frac{v_e}{u_e} = \frac{f_e - v_e}{f_e}}$$

$$\Rightarrow \frac{1}{u_e} = -\frac{27}{50} \Rightarrow u_e = -\frac{50}{27}$$

$$\therefore L = v_0 + |u_e| = 12 \text{ cm}$$

$$12 = v_0 + \left| -\frac{50}{27} \right| = v_0 + \frac{50}{27}$$

$$v_0 = 12 - \frac{50}{27} = \frac{274}{27}$$

Now, for objective $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$ or $\boxed{\frac{v_0}{u_0} = \frac{f_0}{f_0 + u_0}}$

$$\frac{1}{\left(\frac{274}{27}\right)} - \frac{1}{u_0} = \frac{1}{1}$$

$$\frac{1}{u_0} = \frac{24}{274} - 1 \quad \frac{1}{u_0} = \frac{24 - 274}{274}$$

$$\Rightarrow u_0 = -\frac{274}{250} = -1.1 \text{ cm}$$

4. $f_e = 5 \text{ cm}$
 M.P. = -14

In normal adjustment, final image is formed at ∞ .

$$\therefore \text{M.P.} = \frac{-f_0}{f_e} \Rightarrow -14 = \frac{-f_0}{5}$$

$$f_0 = 14 \times 5 = 70 \text{ cm}$$

$$\text{Length of the tube (L)} = f_0 + f_e$$

$$L = 70 + 5 = 75 \text{ cm}$$

5. $\therefore$ Length of telescope

$$L = f_0 + f_e$$

$$\text{Means } 50 = f_0 + f_e \dots(1)$$

Magnifying power

$$M = \frac{-f_0}{f_e} = -9 \text{ or } f_0 = 9 f_e \dots(2)$$

$$\text{from (1) \& (2) } f_0 = 45 \text{ cm \& } f_e = 5 \text{ cm}$$

6. To shift the far point he should use concave lens

$$\text{F.P.} = 200 \text{ cm}$$

$$F = -\text{F.P.} = -200 \text{ cm}$$

7. $F = -\text{F.P.} = -75 \text{ cm}$

$$P = -\frac{100}{75} = -\frac{4}{3} = -1.33 \text{ D}$$

8. $\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{-3} - \frac{1}{(-12)} = \frac{1}{f}$

$$\frac{1}{f} = -\frac{1}{3} + \frac{1}{12} \text{ or } f = -4 \text{ m}$$

9. The person should use convex lens

$$\therefore P = \frac{100}{v} - \frac{100}{u} = \frac{100}{(-50)} - \frac{100}{(-25)}$$

$$\Rightarrow P = -2 + 4 = +2 \text{ D}$$

10. $f_0 = 2 \text{ cm}$, $f_e = 6.25 \text{ cm}$, $L = 15 \text{ cm}$
 $v_e = -25 \text{ cm}$

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow -\frac{1}{25} - \frac{1}{u_e} = \frac{1}{6.25}$$

$$u_e = -5 \text{ cm}$$

Now,

$$L = v_0 + |u_e|$$

$$15 = v_0 + 5 \Rightarrow v_0 = 10 \text{ cm}$$

Now, for objective

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{10} - \frac{1}{u_0} = \frac{1}{2}$$

$$\boxed{u_0 = -2.5 \text{ cm}}$$

11. $f_o = 8\text{mm}$, $f_e = 2.5\text{cm}$, $u_o = -9\text{mm}$, $v_e = 25\text{cm}$
for objective

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} - \left(-\frac{1}{9 \times 10^{-1}}\right) = \frac{1}{8 \times 10^{-1}}$$

$$\frac{1}{v_o} = \frac{10}{8} - \frac{10}{9}$$

$$v_o = 7.2$$

for eyepiece

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{(-25)} - \frac{1}{u_e} = \frac{1}{2.5} = \frac{10}{25}$$

$$-\frac{1}{u_e} = \frac{10}{25} + \frac{1}{25} = \frac{11}{25}$$

$$u_e = -\frac{25}{11} = -2.27\text{ cm}$$

Length of the tube (नली की लम्बाई) = $v_o + |u_e|$
= $7.2 + 2.27 = 9.47\text{ cm}$

12. $M.P. = \frac{v_o}{u_o} \times \left(1 + \frac{D}{f_e}\right)$

$$= -\frac{7.2}{9 \times 10^{-1}} \times \left(1 + \frac{25}{2.5}\right)$$

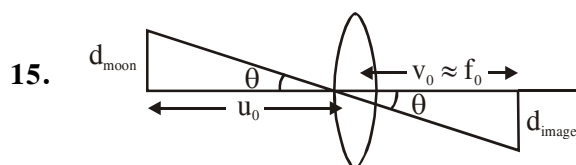
$$M.P. = -88$$

13. $M.P. = -\frac{f_o}{f_e} = -\frac{144}{6}$

$$M.P. = -24$$

Length of the tube = $f_o + f_e$
 $\Rightarrow L = 144 + 6 = 150\text{ cm}$

14. $M.P. = -\frac{f_o}{f_e} = -\frac{1500}{1} = 1500$



since moon is very far. Image will be formed in focal plane

$$\tan\theta = \frac{d_{\text{moon}}}{u_o} = \frac{d_{\text{image}}}{f_o}$$

$$\Rightarrow d_{\text{image}} = \frac{3.48 \times 10^6}{3.8 \times 10^8} \times 1500 = 13.74\text{ cm}$$

16. $f = 5\text{ cm}$, $u = ?$

for closest distance

$$v = -25\text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{5} = \frac{1}{-25} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{u} = -\frac{1}{25} - \frac{1}{5}$$

$$\Rightarrow \boxed{u = -4.2\text{cm}}$$

for farthest distance

$$v' = \infty, u' = ?$$

$$\frac{1}{f} = \frac{1}{v'} - \frac{1}{u'}$$

$$\Rightarrow \frac{1}{5} = \frac{1}{\infty} - \frac{1}{u'}$$

$$\Rightarrow \boxed{u' = -5\text{cm}}$$

17. Maximum angular magnification

$$M = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} \quad M = 6$$

Minimum angular magnification

$$M = \frac{D}{f_e} = \frac{25}{5} = 5$$

18. $A_o = 1\text{mm}^2$
 $u = -9\text{ cm}$

$$f = 10\text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-9)} = \frac{1}{10}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{9} \Rightarrow \frac{1}{v} = \frac{9-10}{90}$$

$$\Rightarrow v = -90\text{ cm}$$

$$\text{Magnification (m)} = \frac{v}{u} = \frac{-90}{-9} = 10$$

19. In image both lengths width gets magnified

$$A_1 = m^2 \times A_o = (10)^2 \times 1 = 100\text{ mm}^2$$

20. Magnifying power = $\frac{D}{u} = \frac{25}{9} = 2.8$

21. Final Image is formed at 25cm

$$v = -25\text{ cm}, f = 10\text{ cm}, u = ?$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-25} - \frac{1}{u} = \frac{1}{10}$$

$$\Rightarrow \frac{1}{u} = -\frac{1}{25} - \frac{1}{10} = \frac{-2-5}{50} \Rightarrow u = -\frac{50}{7} = -7.14\text{ cm}$$

22. Magnification $m = \frac{v}{u} = \frac{25}{7.14} = 3.5$

23. Angular magnification of eye piece (m_e)

$$m_e = \left(\frac{25}{5} \right) = 5 \quad \{\text{for normal adjustment}\}$$

and magnification (m) = $m_0 \times m_e$

$$\Rightarrow m_0 = \frac{m}{m_e} = \frac{-30}{5} = -6$$

$$\text{Also, } m_0 = \frac{f_0 - v_0}{f_0}$$

$$\Rightarrow -6 = \frac{1.25 - v_0}{1.25}$$

$$\Rightarrow v_0 = 8.75 \text{ cm}$$

$$\text{Hence } L = v_0 + f_e = 8.75 + 5 = 13.75 \text{ cm}$$

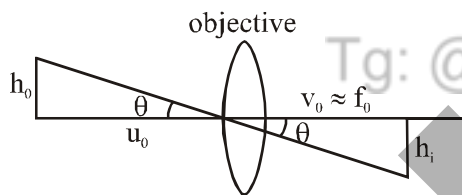
24. $M = -\frac{f_0}{f_e} = -\frac{140}{5} = -28$

25. $M = -\frac{f_0}{f_e} \left(1 + \frac{f_e}{D} \right) = -\frac{140}{5} \left(1 + \frac{5}{25} \right)$

$$M = -33.6$$

26. $L = f_0 + f_e = 140 + 5 = 145 \text{ cm}$

27.



$$\tan \theta = \frac{h_0}{u_0} = \frac{h_i}{f_0}$$

$$h_i = \frac{100}{3000} \times 140$$

$$h_i = 4.7 \text{ cm}$$

Image formed by objective will get magnified by eyepiece

28. $m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$

$$h_1 = 4.7 \text{ cm}$$

$$\text{Now, } m_e = \frac{h_2}{h_1}$$

$$\Rightarrow h_2 = m_e \times h_1 = 6 \times 4.7 = 28 \text{ cm}$$

29. $R_{\text{objective mirror}} = 220 \text{ mm} = R_1$

$$R_{\text{Secondary mirror}} = 140 \text{ mm} = R_2$$

$$f_2 = \frac{R_2}{2} = \frac{140}{2} = 70 \text{ mm}$$

Distance between two mirrors, $d = 20 \text{ mm}$.

When object is at infinity, parallel rays falling on objective mirror on reflection would collect at its focus at

$$f_1 = \frac{R_1}{2} = \frac{220}{2} = 110 \text{ mm}$$

Instead, they fall on secondary mirror at 20 mm from objective mirror $u = f_1 - d$

$$\Rightarrow u_1 = 110 - 20 = 90 \text{ mm}$$

$$\text{From } \frac{1}{v} + \frac{1}{u} = \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f_2} - \frac{1}{u} = \frac{1}{70} - \frac{1}{90} = \frac{9-7}{630} = \frac{2}{630}$$

$$v = \frac{630}{2} = 315 \text{ mm} = 31.5 \text{ cm from } M_2$$

WAVE NATURE OF LIGHT & WAVE OPTICS (INTERFERENCE) : RACE # 01

SOLUTION

1. The distance of a bright fringe from zero order fringe is given by

$$x_n = \frac{n\lambda D}{d}$$

Thus at a given point $n\lambda = \text{constant}$

$$\therefore n_1\lambda_1 = n_2\lambda_2$$

$$n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{16 \times 7200}{4800} = 24$$

$$3. \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$\text{Given that } \frac{I_{\max}}{I_{\min}} = 9$$

$$\therefore \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = 9 \text{ or } \frac{a_1 + a_2}{a_1 - a_2} = 3$$

$$\text{or } a_1 + a_2 = 3a_1 - 3a_2 \text{ or } a_1 = 2a_2$$

Hence answer (4) is correct

Let I_1 and I_2 be the intensities due to two slits.

$$\text{Now } I_1 \propto a_1^2 \text{ and } I_2 \propto a_2^2$$

$$\therefore \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{(2a_2)^2}{a_2^2} = \frac{4}{1}$$

Hence answer (2) is correct

So answer (2) and (4) are correct

$$4. d \sin \theta = \frac{\lambda}{2}$$

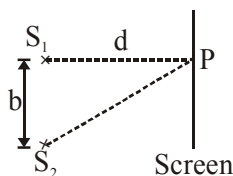
$$\sin \theta = \frac{\lambda}{2d} \text{ (small angle)}$$

$$\theta = \frac{\lambda}{2d} \times \frac{180^\circ}{\pi}$$

$$\theta = \frac{5460 \times 10^{-10}}{2 \times 0.1 \times 10^{-3}} \times \frac{180^\circ}{\pi}$$

$$\theta = 0.156^\circ = 0.16^\circ$$

5. See figure



$$\text{Path difference} = (S_2P - S_1P)$$

$$\text{From figure } (S_2P)^2 - (S_1P)^2 = b^2$$

$$\text{or } (S_2P - S_1P)(S_2P + S_1P) = b^2$$

$$\therefore (S_2P - S_1P) = \frac{b^2}{2d}$$

For dark fringes

$$\frac{b^2}{2d} = (2n+1) \frac{\lambda}{2}$$

$$\text{For } n = 0, \frac{b^2}{2d} = \frac{\lambda}{2} \text{ or } \lambda = \frac{b^2}{d}$$

$$\text{For } n = 1, \frac{b^2}{2d} = \frac{3\lambda}{2} \text{ or } \lambda = \frac{b^2}{3d}$$

$$6. \text{Phase difference} = \frac{2\pi}{\lambda} (\text{Path difference})$$

$$\text{Path difference} = n_1L_1 - n_2L_2$$

$$\text{so phase diff} = \frac{2\pi}{\lambda} (n_1L_1 - n_2L_2)$$

$$7. \beta = \left(\frac{\lambda D}{d} \right)$$

$$\therefore \frac{\beta_1}{\beta_2} = \left(\frac{\lambda_1}{\lambda_2} \right) \left(\frac{D_1}{D_2} \right) \left(\frac{d_2}{d_1} \right)$$

$$\text{or } \frac{D_1}{D_2} = \left(\frac{\beta_1}{\beta_2} \right) \left(\frac{\lambda_2}{\lambda_1} \right) \left(\frac{d_1}{d_2} \right)$$

$$\text{or } \frac{D_1}{D_2} = \frac{1 \times 2 \times 2}{1 \times 1 \times 1} = \frac{4}{1}$$

$$\text{or } D_1 : D_2 = 4 : 1$$

$$8. S = (\mu - 1) t (D/2d)$$

$$S = (5/3 - 1) t (D/2d) = \frac{Dt}{3d}$$

$$9. (\mu - 1)t = 5\lambda$$

$$\lambda = \frac{(\mu - 1)t}{5} = \frac{.5 \times 6 \times 10^{-6}}{5} = 6000 \text{ \AA}$$

10. glass denser medium so phase is change by π

11. $d \sin \theta = n\lambda$

$$\sin \theta = \pm \frac{n\lambda}{d} = \pm \frac{n\lambda}{2.5\lambda}$$

($n = 0, \pm 1, \pm 2$)

Five

15. $(\mu - 1)t = 4\lambda$

$$t = \frac{4\lambda}{(\mu - 1)} = \frac{4 \times 6000 \times 10^{-10}}{.5} = 4.8 \mu\text{m}$$

16. When path difference is λ , then intensity will be maximum.

So maximum intensity is I_0

When path difference is $\frac{\lambda}{3}$

then intensity $I = I_0 \cos^2 \frac{\phi}{2}$

$$= I_0 \cos^2 \left[\frac{1}{2} \times \frac{2\pi}{\lambda} \times \frac{\lambda}{3} \right]$$

$$= I_0 \cos^2 \left(\frac{\pi}{3} \right) = \frac{I_0}{4}$$

17.

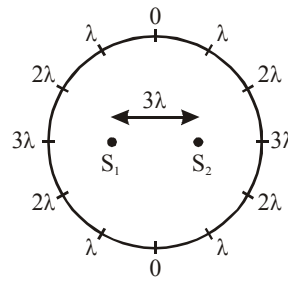


Diagram shows Δx at different point on circle

From diagram it is clear that

number of maxima = 12

**WAVE OPTICS : DIFFRACTION, POLARISATION
& RESOLVING POWER : RACE # 02**

SOLUTION

2. Linear width of principal maximum

$$\frac{2\lambda}{d} \times D$$

According to the given problem

$$d = \frac{2\lambda}{d} \times D \text{ or } D = \frac{d^2}{2\lambda}$$

3. $\beta_0 = 2 \frac{\lambda D}{d}$

$$\therefore \text{Angular width } \theta = \frac{\beta_0}{D} = \frac{2\lambda}{d}$$

$$\text{or } \theta = \frac{2 \times 6328 \text{ \AA}}{0.2 \text{ mm}} \approx 0.36^\circ$$

8. $\mu = \tan 60^\circ = \sqrt{3}$

11. $RL \propto \lambda$

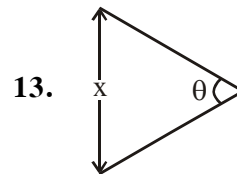
$$\frac{RL_1}{RL_2} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{0.1}{RL_2} = \frac{6000}{4800}$$

$$RL_2 = 0.08 \text{ mm}$$

12. $\theta = \frac{\lambda}{a}$

$$\theta = \frac{5000 \times 10^{-10}}{10 \times 10^{-2}} = 5 \times 10^{-6} \text{ rad}$$



by resolving limit

$$\frac{x}{4 \times 10^5 \times 10^3} = \frac{5000 \times 10^{-10} \times 1.22}{5}$$

$$x = 50 \text{ m}$$

14. For first minimum $a \sin \theta = \lambda$

$$\text{So } \frac{\lambda}{a} = \sin 30^\circ = \frac{1}{2}$$

$$\text{Now for first secondary maximum } a \sin \theta = \frac{3\lambda}{2}$$

$$\text{Therefore } \sin \theta = \frac{3\lambda}{2a} = \frac{3}{2} \left(\frac{1}{2} \right) = \frac{3}{4}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{3}{4} \right)$$

MODERN PHYSICS (PHOTO ELECTRIC EFFECT) : RACE # 01 SOLUTION

1. $E = h\nu = 6.6 \times 10^{-34} \times 10^{15} = 6.6 \times 10^{-19} \text{ J}$
2. $\lambda = 600 \text{ nm} = 6000 \text{ \AA}$
 $I = 100 \text{ W/m}^2$
 $I = n h\nu$
 $n = \text{Number of photons per second per unit area}$

$$I = \frac{nhc}{\lambda}$$

$$n = \frac{I\lambda}{hc} = I\lambda \times 5 \times 10^{24}$$

$$= 100 \times 600 \times 10^{-9} \times 5 \times 10^{24}$$

$$= 3 \times 10^{20} \text{ photons/sec/m}^2$$

$$1 \text{ m}^2 \text{ — } 3 \times 10^{20} \text{ photons / sec}$$

$$1 \text{ cm}^2 \text{ — } 3 \times 10^{16} \text{ photons / sec}$$
3. $\lambda_1 = 300 \text{ nm}$
 $\lambda_2 = 600 \text{ nm}$
 $\phi_0 \propto \frac{1}{\lambda}$

$$\frac{\phi_{01}}{\phi_{02}} = \frac{600 \times 10^{-9}}{300 \times 10^{-9}} = \frac{2}{1}$$
4. The stopping potential (V_0) depends on the frequency (ν) of incident light as :

$$eV_0 = h\nu - \phi_0$$

 If $\nu \uparrow$ then $V_0 \uparrow$
5. Intensity $\propto$ No. of photons $\propto$ No. of photo electrons.
7. $E = 3.8 \text{ eV}$
 $\phi_0 = 2.8 \text{ eV}$
 $K.E._{\text{max}} = E - \phi_0 = 1 \text{ eV}$
 it means energy of electron should be in between 0 to 1 eV.
8. $n \propto \frac{1}{d^2}$

$$\frac{n_2}{n_1} = \frac{d_1^2}{d_2^2} \Rightarrow \frac{n_2}{n_1} = \frac{d^2}{(d/2)^2}$$

$$\Rightarrow \frac{n_2}{n_1} = 4$$

$$\Rightarrow n_2 = 4n_1$$

 Note :- The maximum K.E does not depend on the distance.
9. $v_{\text{max}} = 1.2 \times 10^6 \text{ m/s}$

$$\frac{e}{m} = \text{specific charge} = 1.8 \times 10^{11} \text{ C/kg}$$

$$K.E = \frac{1}{2}mv^2$$

$$eV_0 = \frac{1}{2}mv^2$$

$$V_0 = \frac{1}{2} \frac{m}{e} v^2$$

$$= \frac{1}{2} \times \frac{1}{1.8 \times 10^{11}} \times (1.2 \times 10^6)^2$$

$$V_0 = 4 \text{ volt}$$

10. $eV_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$
 as $\lambda \downarrow$ $V_0 \uparrow$
11. $\frac{E_2}{E_1} = \frac{\lambda_1}{\lambda_2} = \frac{10000 \text{ \AA}}{5000 \text{ \AA}} = 2$
 $E_2 = 2E_1 = 2 \times 1.23 \text{ eV} = 2.46 \text{ eV}$
 $V_0 = 1.36 \text{ V}$
 $(K.E.)_{\text{max}} = eV_0 = 1.36 \text{ eV}$

$$\phi_0 = 2E_1 - (K.E.)_{\text{max}}$$

$$= 2.46 - 1.36 = 1.10 \text{ eV}$$
12. $V_0 = 4 \text{ V}$
 $K_{\text{max}} = eV_0$
 $K_{\text{max}} = 4 \text{ eV}$
13.
$$K_{\text{max}} = \frac{hC}{\lambda} - \phi$$

$$K_A = \frac{hC}{\lambda_A} - \phi \quad \dots(1)$$

$$K_B = \frac{hC}{\lambda_B} - \phi \quad \dots(2)$$

$$\therefore \lambda_A = 2\lambda_B$$

 from eqn (1)

$$K_A = \frac{hC}{2\lambda_B} - \phi$$

$$K_A = \frac{1}{2} \left(\frac{hC}{\lambda_B} \right) - \phi \quad \dots(3)$$

 from eqn (2) & (3)

$$K_A = \frac{1}{2} (K_B + \phi) - \phi$$

$$K_A = \frac{K_B}{2} - \frac{\phi}{2}$$

$$K_A < \frac{K_B}{2}$$

$$1. \quad \lambda = \frac{h}{p} \Rightarrow \lambda \propto \frac{1}{p}$$

$$2. \quad \lambda = \frac{h}{p}$$

$$4. \quad \lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

$$\frac{\lambda p}{\lambda \alpha} \propto \sqrt{\frac{m \alpha}{m p}} = \frac{2}{1}$$

$$5. \quad \lambda = \sqrt{\frac{150}{V}} \text{ \AA}$$

$$\text{Here } \lambda = 1 \text{ \AA} \\ \therefore V = 150 \text{ Volt}$$

$$6. \quad \lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda \propto \frac{1}{\sqrt{E}}$$

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{E_1}{E_2}} = \sqrt{\frac{1}{2}}$$

$$\lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

$$7. \quad \lambda = \frac{h}{mV} = \frac{6.6 \times 10^{-34}}{1 \times 2000} = 3.3 \times 10^{-37} \text{ m} \\ = 3.3 \times 10^{-27} \text{ \AA}$$

$$8. \quad \lambda_{\text{electron}} = \lambda_{\text{photon}}$$

$$E_{\text{ph}} = \frac{hc}{\lambda_{\text{ph}}}, p_e = \frac{h}{\lambda_e}$$

$$\frac{E_{\text{ph}}}{p_e} = \frac{hc}{\lambda_{\text{ph}}} \times \frac{\lambda_e}{h} = c$$

$$10. \quad n = 4 \\ r_n = (53n^2) \text{ pm} \\ 2\pi r_n = n\lambda \\ 2 \times 3.14 \times 53 \times 10^{-12} \times (4)^2 = 4 \times \lambda \\ \lambda = 1.33 \text{ nm}$$

$$11. \quad \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m}} \times \frac{1}{\sqrt{E}}$$

$$\log \lambda = \log \frac{h}{\sqrt{2m}} + \log \frac{1}{\sqrt{E}}$$

$$\log \lambda = \log \frac{h}{\sqrt{2m}} - \frac{1}{2} \log E$$

$$\log \lambda = -\frac{1}{2} \log E + \log \frac{h}{\sqrt{2m}}$$

$$y = m x + C$$

$$m = -ve$$

$$C = +ve$$

2. ${}_{90}\text{X}^{200} \rightarrow {}_{80}\text{Y}^{168}$

$$A_I = 200 \quad A_F = 168$$

$$Z_I = 90 \quad Z_F = 80$$

$$N_\alpha = \frac{A_I - A_F}{4} = \frac{200 - 168}{4} = \frac{32}{4} = 8$$

$$N_\beta = Z_F - [Z_I - 2N_\alpha]$$

$$= 80 - [90 - 2 \times 8] = 80 - 74 = 6$$

5. ${}_{13}^{27}\text{Al} + {}_2^4\text{He} \rightarrow {}_{15}^{30}\text{P} + {}_0^1\text{n}^1$

6. $N_0 = 10^6$

$$T_H = 20 \text{ sec}$$

$$t = 10 \text{ sec.}$$

$$N = N_0 \times \left(\frac{1}{2}\right)^{t/T_H} = 10^6 \times \left(\frac{1}{2}\right)^{10/20}$$

$$= \frac{10^6}{\sqrt{2}} = \frac{10^6}{1.414} = 7 \times 10^5$$

7. $T_p = 10 \text{ min}$ $T_Q = 15 \text{ min}$

$$\text{at } t = 0, N_{OP} = N_{OQ}$$

$$\frac{N_P}{N_Q} = \frac{N_{OP} \times \left(\frac{1}{2}\right)^{t/T_p}}{N_{OQ} \times \left(\frac{1}{2}\right)^{t/T_Q}} = \frac{N_0 \times \left(\frac{1}{2}\right)^{30/10}}{N_0 \times \left(\frac{1}{2}\right)^{30/15}}$$

$$= \frac{\left(\frac{1}{2}\right)^3}{\left(\frac{1}{2}\right)^2} = \frac{1}{2}$$

8. $T_{\text{avg}} = \frac{1}{\lambda}$

$$\frac{N}{N_0} = e^{-\lambda \times T_{\text{avg}}} \Rightarrow \frac{N}{N_0} = e^{-\lambda \times \frac{1}{\lambda}} \Rightarrow \frac{N}{N_0} = \frac{1}{e}$$

$$\text{decay fraction} = 1 - \frac{N}{N_0} = \frac{N'}{N_0} = 1 - \frac{1}{e} = \frac{e-1}{e}$$

9. $\lambda_1 = 10\lambda$ & $\lambda_2 = \lambda$

$$t = 0 \Rightarrow N_{01} = N_{02} = N_0$$

$$\therefore N = N_0 e^{-\lambda t}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}}$$

$$\frac{1}{e} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} \Rightarrow \frac{1}{e} = e^{-9\lambda t} = \frac{1}{e^{9\lambda t}}$$

$$e^{9\lambda t} = e^1 \Rightarrow 1 = 9\lambda t \Rightarrow \boxed{t = \frac{1}{9\lambda}}$$

10. $T_H = 3.8 \text{ days}$

$$\frac{N}{N_0} = \frac{1}{20}$$

$$N = N_0 e^{-\lambda t}$$

$$t = \frac{2.303}{\lambda} \log_{10} \left(\frac{N_0}{N} \right)$$

$$t = \frac{2.303}{0.693} \times T_H \log_{10} \left(\frac{20}{1} \right)$$

$$t = \frac{2.303}{0.693} \times 3.8 [\log_{10} 2 + \log_{10} 10 - \log_{10} 1]$$

$$t = \frac{2.303}{0.693} \times 3.8 [1.3010]$$

$$t = 16.42 \text{ Days}$$

11. $\frac{dN}{dt} = \lambda N$

at 4min $\frac{dN}{dt} = 322$ counts/sec

at 36min $\frac{dN}{dt} = 161$ counts/sec

$$\frac{322}{161} = \frac{\lambda \times N_1}{\lambda \times N_2} \Rightarrow \boxed{\frac{N_1}{N_2} = 2}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{-\lambda \times 4}}{N_0 e^{-\lambda \times 36}}$$

$$2 = e^{-4\lambda} \times e^{+36\lambda}$$

$$2 = e^{32\lambda}$$

$$\log_e 2 = 32\lambda \Rightarrow \boxed{\frac{0.693}{32} = \lambda}$$

$$T_H = \frac{0.693}{\lambda} = \frac{0.693}{0.693} \times 32 = 32 \text{ min}$$

OR

$$R = \lambda N$$

$$\frac{R_1}{R_2} = \frac{\lambda N_1}{\lambda N_2} \quad \& \quad \frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T_H}$$

$$\frac{R_1}{R_2} = \left(\frac{1}{2}\right)^{\frac{4}{T} - \frac{36}{T}} \Rightarrow \frac{322}{161} = \frac{2}{1} = \left(\frac{1}{2}\right)^{-\frac{32}{T}}$$

$$T = 32 \text{ min.}$$

12. $T_H = 20 \text{ min}$

$$\frac{N_1}{N_0} = \frac{67}{100} \quad \& \quad \frac{N_2}{N_0} = \frac{33}{100}$$

$$\therefore \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$$

$$\frac{67}{100} = \left(\frac{1}{2}\right)^{t_1/20} \quad \dots (1)$$

$$\frac{33}{100} = \left(\frac{1}{2}\right)^{t_2/20} \quad \dots (2)$$

from equation (1) & (2)

$$\Rightarrow \frac{33}{100} \times \frac{100}{67} = \frac{\left(\frac{1}{2}\right)^{t_2/20}}{\left(\frac{1}{2}\right)^{t_1/20}}$$

$$\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^{\frac{t_2 - t_1}{20}} \Rightarrow t_2 - t_1 = 20 \text{ min}$$

13. $t = 0$ & $A_0 = 1600$ counts/sec

$$t = 8 \text{ sec} \quad \& \quad A = 100 \text{ counts/sec} \quad \left| \quad A = 1600 \times \left(\frac{1}{2}\right)^{6/2}$$

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{t/T_H}$$

$$= 1600 \times \frac{1}{8}$$

$$\frac{100}{1600} = \left(\frac{1}{2}\right)^{8/T_H}$$

$$= 200 \text{ counts/sec}$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{8/T_H}$$

$$\boxed{T_H = \frac{8}{4} = 2 \text{ sec}}$$

16. $N = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = N_0 e^{-\lambda t} (-\lambda)$$

or $-\frac{dN}{dt} = N\lambda$

$$\text{Activity} = \left| -\frac{dN}{dt} \right| = N\lambda$$

i.e. Activity is independent of time.

18. $A = N\lambda$

$$\frac{A_1}{A_2} = \frac{N_1 \lambda_1}{N_2 \lambda_2}$$

$$\frac{A_1}{A_2} = \frac{N_1}{N_2} \times \frac{T_{H_2}}{T_{H_1}} \quad \left(\because T_H \propto \frac{1}{\lambda} \right)$$

$$\frac{A_1}{2A_2} = \frac{2N}{N} \times \frac{T_{H_2}}{T_{H_1}}$$

$$\frac{T_{H_1}}{T_{H_2}} = 4$$

$$1. \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2} \right)^{1/3} = \left(\frac{135}{40} \right)^{1/3} = 1.50$$

$$2. \quad \frac{v_1}{v_2} = \frac{2}{1}$$

According to law of momentum conservation

$$P_1 = P_2$$

$$m_1 v_1 = m_2 v_2$$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

As nuclear density is constant and mass $\propto r^3$

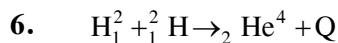
$$\left(\frac{r_1}{r_2} \right)^3 = \left(\frac{v_2}{v_1} \right)$$

$$\frac{r_1}{r_2} = \left(\frac{v_2}{v_1} \right)^{1/3}$$

$$\frac{r_1}{r_2} = \left(\frac{1}{2} \right)^{1/3}$$

$$\frac{r_1}{r_2} = \frac{1}{2^{1/3}}$$

4. More $\frac{\text{B.E.}}{A}$ means more stability.



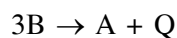
$$\frac{\text{B.E.}}{A} \text{ is } {}_1^2\text{H} \rightarrow x_1 \text{ and } {}_2^4\text{He} \rightarrow x_2$$

$$Q = 4x_2 - 4x_1 = 4(x_2 - x_1)$$

$$7. \quad \text{B.E. of } A \rightarrow E_a$$

$$\text{B.E. of } B \rightarrow E_b$$

given nuclear reaction



$$Q = E_a - 3E_b$$

$$\text{Because } Q = (\text{B.E.})_{\text{Final}} - (\text{B.E.})_{\text{Initial}}$$

$$8. \quad \left(\frac{\text{B.E.}}{A} \right)_x = 8.5 \text{ MeV} \quad \& \quad \left(\frac{\text{B.E.}}{A} \right)_U = 7.6 \text{ MeV}$$

$$\left(\frac{\text{B.E.}}{A} \right)_y = 8.5 \text{ MeV}$$

Q = energy liberated

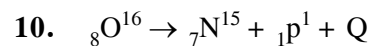
$$= 2 \times 8.5 \times 117 \text{ MeV} - 236 \times 7.6 \text{ MeV}$$

$$= 195.4 \approx 200 \text{ MeV}$$

$$\begin{aligned} 9. \quad \Delta m &= ZM_p + (A-Z)M_n - m({}_6\text{C}^{12}) \\ &= 6 \times 1.007825 + 6 \times 1.008665 - 12 \\ &= 12.09894 - 12 \\ &= 0.9894 \text{ amu} \end{aligned}$$

$$\text{B.E.} = 0.9894 \times 931 \text{ MeV} = 92.11 \text{ MeV}$$

$$\text{B.E per nucleon} = \frac{\text{B.E.}}{A} = \frac{92.11}{12} = 7.676 \text{ MeV}$$



$$\begin{aligned} \Delta m &= m({}_8\text{O}^{16}) - [m({}_7\text{N}^{15}) + m_p] \\ &= 15.994915 - [15.000108 + 1.007825] \\ &= 15.994915 - 16.00793 = -0.013018 \text{ amu} \end{aligned}$$

$$Q = -0.013018 \times 931 \text{ MeV} \approx -12.13 \text{ MeV}$$

Note :- Here negative sign indicates that the reaction is endothermic.

$$11. \quad \Delta m = 1\text{gm} - 0.993 \text{ gm}$$

$$= 0.007 \text{ gm} = 7 \times 10^{-6} \text{ kg}$$

$$E = \Delta mc^2 = 7 \times 10^{-6} \times 9 \times 10^{16} = 63 \times 10^{10} \text{ J}$$

13. B.E of products > BE of reactants

14. released energy

$$= (\text{BE. of product}) - (\text{BE of reactant})$$

if (BE. of product) > (BE of reactant) then energy released

$$18. \quad \frac{R_N}{R_{\text{He}}} = 14^{1/3}$$

$$\therefore R \propto A^{1/3}$$

$$\left(\frac{A_N}{A_{\text{He}}} \right)^{1/3} = 14^{1/3}$$

$$\frac{A_N}{A_{\text{He}}} = 14$$

$$A_N = 14 \times 4 = 56$$

$$N+P = 56$$

$$\text{atomic number} = P = 56 - 30 = 26$$

1. CKT $\Rightarrow$ A

Both diode are FB $\Rightarrow$

$$\text{So Req.} = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 4}{4 + 4} = 2\Omega$$

$$\text{So } I = V/R = 8/2 = 4 \text{ amp}$$

CKT $\Rightarrow$ B

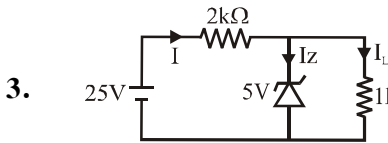
$$\text{Req.} = 4\Omega$$

$$I = 8/4 = 2 \text{ amp}$$

2.

F.B. so diode Resistance $\Rightarrow 25\Omega$

$$\begin{aligned} I &= 5/35 \\ &= 1/7 \text{ Amp.} \end{aligned}$$

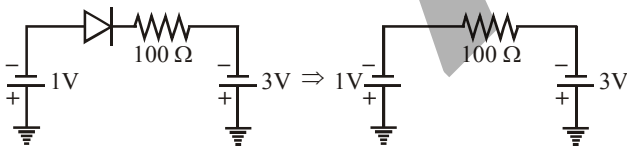


$$I_L = \frac{V_Z}{R_L} = \frac{5}{1k\Omega} = 5 \text{ mA}$$

$$I = \frac{V - V_Z}{R_s} = \frac{25 - 5}{2k\Omega} = 10 \text{ mA}$$

$$I_L = I - I_Z = 10 - 5 = 5 \text{ mA}$$

5. Diode is forward Bias so



$$I = \frac{3 - 1}{100} = \frac{2}{100} \Rightarrow 20 \text{ mA}$$

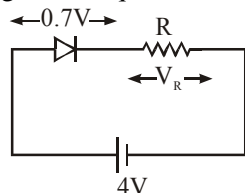
6. Reverse bias resistance = $\frac{\Delta V}{\Delta I}$

$$= \frac{1V}{0.5\mu A} = \frac{1V}{0.5 \times 10^{-6} A} = 2 \times 10^6 \Omega$$

7. By drawing a circuit as given in question then it will be as follows

From this circuit, using KVL, we have

$$4V = 0.7V + V_R$$

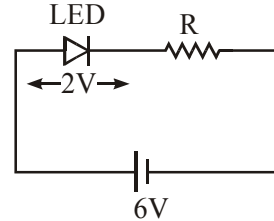


$$= 0.7V + IR$$

$$\Rightarrow IR = 4 - 0.7 = 3.3 \text{ volt}$$

$$\Rightarrow R_{\min} = \frac{3.3 \text{ volt}}{I_{\max}} = \frac{3.3 \text{ volt}}{5 \text{ mA}} = \frac{3.3V}{5 \times 10^{-3} A} = 660 \Omega$$

9. Given question is equivalent to the following circuit.



$$\text{Using KVL, } 2 + IR = 6$$

$$R = \frac{6 - 2}{I} = \frac{4V}{10\mu A}$$

$$\Rightarrow R = 4 \times 10^5 \Omega = 400 \text{ K}\Omega$$

10. For intrinsic semiconductor,

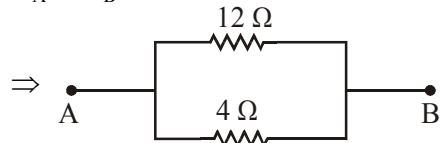
$$n_e = n_h = n_i = 7.07 \times 10^{15} \text{ m}^{-3}$$

In extrinsic (doped) semiconductor,

$$n_e n_h = n_i^2$$

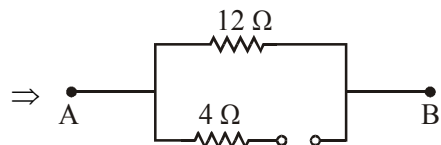
$$\Rightarrow \frac{n_e^2}{n_h} = \frac{(7.07 \times 10^{15})^2}{5 \times 10^{22}} = 10^9 \text{ m}^{-3}$$

13. (i) $V_A > V_B \Rightarrow$ diode is in FB



$$\Rightarrow R_{AB} = 3\Omega$$

(ii) $V_A < V_B \Rightarrow$ diode is in RB



$$\Rightarrow R_{AB} = 12\Omega$$

1. (1) $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2\text{mA}}{15\mu\text{A}} = 133.33$

(2) $\Delta V_{BE} = R_{in} \Delta I_B$
 $= 665 \times 15 \times 10^{-6} \text{ V}$

$g_m = \frac{I_{out}}{V_{in}} = \frac{\Delta I_C}{\Delta V_{BE}} = 0.2 \Omega^{-1}$

(3) $A_v = \beta \frac{R_o}{R_{in}} = 1000$

2. $\alpha = 0.95$ $I_E = 10 \text{ mA}$

(1) $\alpha = I_C / I_E$

(2) $I_E = I_B + I_C$

(3) $\beta = \frac{I_C}{I_B}$

3. $A_p = \alpha A_v$

$\alpha = \frac{A_p}{A_v} = \frac{800}{840}$

and $\beta = \frac{\alpha}{1-\alpha} \approx 19.83$

Also, $\beta = \frac{I_C}{I_B}$

$\Rightarrow \boxed{I_C = \beta \times I_B}$

4. $\beta = \frac{I_C}{I_B}$ & $I_E = I_B + I_C$

5. $I_C = \beta I_B$
 $= 50 \times 40 \times 10^{-6}$
 $= 2 \times 10^{-3} \text{ A}$

$V_{CC} = I_C R_C + V_{CE}$

$V_{CE} = V_{CC} - I_C R_C$
 $= 10 - (2 \times 10^{-3}) \times 4 \times 10^3$
 $= 2 \text{ V}$

7. $\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$

8. $\alpha = 0.98$, and in CE mode, $R_L = 5 \text{ K}\Omega$ and
 $R_{in} = R_{BE} = 70 \Omega$

For CE mode, $A_C = \beta = \frac{\alpha}{1-\alpha} = 49$

$A_v = \beta \frac{R_L}{R_{in}} = 49 \times \frac{5000}{70} = 3500$

and $A_p = A_v \times A_C = 3500 \times 49$
 $= 1.715 \times 10^5$

12. $V_{out} = (A_v \times 0.004 \text{ V}) \sin(\omega t + \pi/2 \pm \pi)$
 [because CE amplifier produces a phase difference of π]

14. (i) $A_v = \alpha \frac{R_{out}}{R_{in}} = 0.98 \times \frac{100 \text{ k}\Omega}{200 \Omega} = 490$

(ii) Use $A_p = A_v \times A_C$, $A_p = 490 \times 0.98 = 480$

(iii) Use, $\frac{I_C}{I_B} = \beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$

$I_B = \frac{I_C}{49} = 0.04 \text{ mA}$ ($\because I_C = 1.96 \text{ mA}$)

4. $Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

5. $Y = (A + B) \cdot B$
 $= A \cdot B + B \cdot B$
 $= A \cdot B + B$
 $= B (A + 1) = B$

6. $Y = (A \cdot B) \cdot (\overline{A} \cdot B)$ ($A \cdot \overline{A} = 0$)
 $= 0$

7. $Y = \overline{(\overline{A \cdot A \cdot B}) \cdot (\overline{B \cdot A \cdot B})}$
 $= \overline{A \cdot (\overline{A \cdot B}) + B \cdot (\overline{A \cdot B})}$
 $= A \cdot (\overline{A \cdot B}) + B \cdot (\overline{A \cdot B})$
 $= A \cdot (\overline{A} + \overline{B}) + B \cdot (\overline{A} + \overline{B})$
 $= A \cdot \overline{A} + A \cdot \overline{B} + B \cdot \overline{A} + B \cdot \overline{B}$
 $= A \cdot \overline{B} + B \cdot \overline{A}$ XOR gate

9. $Y = \overline{A \cdot B}$ Nand gate

10. $Y = \overline{A} \cdot \overline{B} + A \cdot B$